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Quantum Fundamental Speed Theory: Mathematical Derivations of Dark Matter and Dark Energy from the Speed Field

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Abstract

We present a quantum formulation of the Fundamental Speed Theory (QFST) built from the classical FST Lagrangian, where the dimensionless speed field v^μ plays a dual role: it modifies gravity classically in galaxies, while its quantum excitations behave as ultra-light dark matter on cosmological scales.

On galactic scales, the classical FST equation fits rotation-curve data without dark matter (171 SPARC galaxies, $\chi^2_\nu = 0.170$). Linearization yields a screening scale $\mu = 1/\lambda_{\text{screen}}$, implying a particle mass $m = \hbar/(c\,\lambda_{\text{screen}}) = 3.88 \times 10^{-24}$ eV, with no free parameters. Quantum corrections to the galactic equation are suppressed by $(\ell_{\text{Pl}}/\lambda_{\text{screen}})^2 \approx 10^{-103}$, ensuring classical-quantum separation.

Dark energy arises from the logarithmic potential $V(\tilde{v}) = -V_0 \ln\left(1 + \tilde{v}^2/v_0^2\right)$. We obtain a positive vacuum energy density $\rho_\Lambda = [c^4/(16\pi G L_0^2)] \cdot U(\tilde{v}_{\text{now}})$ and determine $\tilde{v}_{\text{now}} = 8.21 \times 10^{-12}$ by matching the observed ρ_Λ. At the background level, QFST reproduces the standard expansion history as tested by Cosmic Chronometer $H(z)$ data. Full discrimination from ΛCDM requires a Boltzmann-code implementation at the perturbation level.

In a spatially flat FLRW spacetime, we derive the reduced kinetic scalar including Hv terms, construct the stress-energy tensor $T^{(v)}_{\mu\nu}$, and obtain closed-form expressions for ρ_v and p_v. The equation of state $w(z) = p_v/\rho_v$ is determined by the coupled background system (Friedmann equations plus the $v(t)$ equation of motion). Detailed derivations and dimensional checks are provided in the appendices.

Article Info

Keywords:

Quantum Fundamental Speed Theory, Dark Matter, Dark Energy, Ultralight Dark Matter, Modified Gravity, Logarithmic Potential, Screening Length, Quantum Suppression, Schrödinger–Poisson System, FLRW Cosmology.

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1. NOTATION

- $c = 2.99792458 \times 10^8$ m/s: speed of light
- $\hbar = 1.054571817 \times 10^{-34}$ J·s: reduced Planck constant
- $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}: Newton's gravitational constant
- $\ell_{\text{Pl}} = \sqrt{\hbar G/c^3} = 1.616 \times 10^{-35}$ m: Planck length
- $L_0 = 10$ kpc $= 3.08567758 \times 10^{20}$ m: characteristic galactic scale
- $\lambda_{\text{screen}} = 1.647$ pc $= 5.082 \times 10^{16}$ m: screening length
- $v_0 = 0.911 \times 10^{-3}$: asymptotic field value (measured in galaxies)
- $v_{\text{now}} = 7.48 \times 10^{-15}$: present cosmic vacuum field value (derived in Section 9)
- $\tilde{v}_{\text{now}} = v_{\text{now}}/v_0 = 8.21 \times 10^{-12}$: dimensionless present cosmic vacuum field value
- $\beta_{\text{eff}} = 2.0 \times 10^7$: effective coupling constant
- $c_1 = 0.51$, $c_2 = -0.07$, $c_3 = 0.32$: kinetic coefficients
- $m = 3.88 \times 10^{-24}$ eV: mass of QFST quanta (derived in Section 8)

1.1. Definition of the Speed Field

The fundamental entity of the Fundamental Speed Theory (FST) is the dimensionless vector field v^μ. We refer to it as the **speed field** for the following reasons.

1.1.1. Dimensional Definition: Ratio to the Speed of Light

The field ν^μ is defined as the ratio of a characteristic velocity to the speed of light:

$$\nu^\mu \equiv \frac{V^\mu}{c}, \tag{1}$$

where V^μ is a velocity field with dimensions $[L\,T^{-1}]$ and c is the speed of light. Consequently, ν^μ is dimensionless.

In the asymptotic region far from all sources, the field approaches the vacuum value

$$\nu^\mu \to (\nu_0, 0, 0, 0), \qquad \nu_0 = 0.911 \times 10^{-3}. \tag{2}$$

This ratio is fixed by galactic-scale kinematics rather than chosen arbitrarily. From the unified acceleration scale of FST [1],

$$A_0 = \frac{(c_1 + c_3)\,\nu_0^2\,c^2}{L_0} \simeq 2.01 \times 10^{-10}\,\text{m/s}^2, \tag{3}$$

we obtain the characteristic galactic velocity scale

$$v_{\text{char}} = \sqrt{A_0 L_0} \simeq 249\,\text{km/s}. \tag{4}$$

Thus,

$$\nu_0 = \frac{v_{\text{char}}}{c} \simeq 8.3 \times 10^{-4}, \tag{5}$$

which is consistent with the empirically fitted order-of-magnitude $\nu_0 \sim 10^{-3}$ used throughout the manuscript.

The speed field is therefore a direct measure of galactic-scale velocities relative to the cosmic speed limit.

1.1.2. Why "Speed Field" and Not "Aether"?

A potential point of confusion is the interpretation of ν^μ. In this framework, ν^μ is:

i. **Not a conventional velocity field** of matter. It encodes the intensity and directionality of organized motion at each spacetime point.
ii. **Not an "aether" field.** Unlike Einstein–Aether models [2], which impose a fixed unit-norm constraint $\nu^\mu \nu_\mu = -1$ via a Lagrange multiplier, FST imposes no such constraint. The asymptotic value ν_0 is an empirical boundary condition determined from galactic rotation curves [1], not a theoretical postulate.
iii. **A kinetic field.** The field represents motion as a primary ontological entity. Matter, forces, and spacetime itself are emergent manifestations of this field and its gradients [3].

The name "speed field" reflects the fact that ν^μ is a dimensionless speed-like quantity, and that its gradients—not the field itself—generate the physical effects we perceive as gravity, inertia, and spacetime curvature.

1.1.3. Connection to Spacetime

As proven in the companion paper [3], the spacetime metric $g_{\mu\nu}$ emerges from gradients of the speed field:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - c_1\nu_0^2 \int G_{\text{ret}}(x - x')(\partial'_\mu\phi)(\partial'_\nu\phi)\,d^4x' + \mathcal{O}(h^2). \tag{6}$$

This relation establishes that ν^μ is not merely a field in spacetime; it is the field from which spacetime itself is constructed.

2. INTRODUCTION

The Fundamental Speed Theory (FST) was introduced as a classical vector-tensor theory of gravity to explain galactic rotation curves without particle dark matter [1]. Its central equation on galactic scales is

$$\frac{d^2\tilde{\nu}}{d\xi^2} + \frac{2}{\xi}\frac{d\tilde{\nu}}{d\xi} = \beta_{\text{eff}}\tilde{\nu}^3,$$

where $\tilde{\nu} = \nu/\nu_0$ is the normalized field and $\beta_{\text{eff}} = 2.0 \times 10^7$. This single equation successfully describes 171 SPARC galaxies with a mean reduced chi-squared of $\chi^2_\nu = 0.170$ [4].

The goal of the present paper is to provide the quantum completion of this framework (QFST) and to show how the same ν-field yields a quantum dark-matter sector and a dynamical dark-energy sector derived from the FST Lagrangian. Our emphasis is therefore on first-principles derivations (quantization, non-relativistic limit, screening length, and the logarithmic potential) together with background-level cosmological consistency checks. Decisive observational discrimination from ΛCDM is expected at the perturbation level (matter power spectrum and CMB anisotropies), which requires a dedicated Boltzmann-code implementation.

In addition, the companion "spacetime fabric" analysis demonstrated that the metric $g_{\mu\nu}$ emerges from the speed field ν^μ and its gradients [3]:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - c_1\nu_0^2 \int G_{\text{ret}}(x - x')(\partial'_\mu\phi)(\partial'_\nu\phi)\,d^4x' + \mathcal{O}(h^2),$$

establishing that spacetime is not an independent background but rather emerges from the speed field itself.

2.1. Clarification on the Nature of Dark Matter

A clarification is necessary at the outset regarding the relationship between classical FST and its quantum counterpart. Classical FST explains galactic rotation curves without any need for dark matter. QFST, on the other hand, predicts the existence of quanta of the ν field. These two statements are not contradictory because they apply to different scales:

- On galactic scales ($r \sim$ kpc), quantum corrections to the classical field equation are suppressed by a factor of 1.01×10^{-103} (proved in Section 7). The classical FST equation alone determines galactic dynamics. The quanta are present, but their gravitational effect is entirely negligible.
- On cosmological scales ($r \sim$ Mpc and larger), the cumulative effect of many quanta over cosmic time becomes significant. Their collective behavior, described by the Schrödinger-Poisson system, mimics what astronomers call dark matter.
- Thus, dark matter is not an exotic particle postulated ad hoc. It is the quantum manifestation of the same ν field that modifies gravity classically. The classical field dominates on galactic scales, while its quanta dominate on cosmological scales.

This resolution is possible because the two phenomena operate on vastly different scales, separated by a factor of 10^{103} in quantum correction magnitude.

The paper is organized as follows. Section 3 reviews the classical FST Lagrangian and the emergence of spacetime. Section 4 performs canonical quantization. Section 5 presents the non-relativistic limit and the Schrödinger-Poisson system. Section 6 discusses soliton solutions and structure formation. Section 7 proves the suppression of quantum effects on galactic scales. Section 8 derives the mass of QFST quanta. Section 9 derives dark energy from the logarithmic potential. Section 10 presents predictions and observational constraints. Section 12 concludes. Detailed appendices provide all step-by-step derivations.

3. CLASSICAL FST AND SPACETIME EMERGENCE

3.1. The Fundamental Action

The FST action is [1]:

$$S = \int d^4x\sqrt{-g}\left[\frac{c^4}{16\pi G}R + \mathcal{L}_V(\nu, \nabla\nu) + \mathcal{L}_m\right] \tag{7}$$

3.2. The Vector Field Lagrangian Density

The vector field Lagrangian density is [1]:

$$\mathcal{L}_V = \frac{c^4}{16\pi G L_0^2}\left[-\frac{1}{2}\mathcal{K}(\nabla\nu) - V(\tilde{\nu})\right] \tag{8}$$

where $L_0 = 10$ kpc is the characteristic galactic scale.

3.3. The Kinetic Term

The kinetic term is [1]:

$$\begin{aligned}\mathcal{K}(\nabla\nu) &= c_1 L_0^2(\nabla_\mu\nu_\nu)(\nabla^\mu\nu^\nu) + c_2 L_0^2(\nabla_\mu\nu^\mu)^2 \\ &\quad + c_3 L_0^2(\nabla_\mu\nu_\nu)(\nabla^\nu\nu^\mu)\end{aligned} \tag{9}$$

with $c_1 = 0.51$, $c_2 = -0.07$, $c_3 = 0.32$, determined from stability and cosmological constraints.

3.4. The Logarithmic Potential

The non-perturbative potential entering the vector-sector Lagrangian is [1]:

$$\boxed{V(\tilde{\nu}) = -V_0 \ln\left(1 + \frac{\tilde{\nu}^2}{\nu_0^2}\right)} \tag{10}$$

where $\nu_0 = 0.911 \times 10^{-3}$ is the asymptotic vacuum value of the field. Because the vector-sector Lagrangian density appears as $-\frac{1}{2}\mathcal{K} - V$ in Eq. (8), it is useful to define the corresponding non-negative potential-energy function

$$U(\tilde{\nu}) \equiv -V(\tilde{\nu}) = V_0 \ln\left(1 + \frac{\tilde{\nu}^2}{\nu_0^2}\right) \geq 0. \tag{11}$$

The late-time vacuum energy density is proportional to $U(\tilde{\nu}_{\text{now}})$, ensuring $\rho_\Lambda > 0$ in the conventions adopted here.

3.5. The Metric-Field Relation

From the spacetime fabric paper [3]:

$$\boxed{\begin{aligned}g_{\mu\nu}(x) &= \eta_{\mu\nu} - c_1\nu_0^2 \int G_{\text{ret}}(x-x')(\partial'_\mu\phi)(\partial'_\nu\phi)\, d^4x' \\ &\quad + \mathcal{O}(h^2)\end{aligned}} \tag{12}$$

This relation demonstrates that spacetime emerges from the speed field and its gradients.

3.6. The Screening Length

From the linearized field equation [1]:

$$\lambda_{\text{screen}} = \frac{L_0}{\sqrt{\frac{3\beta_{\text{eff}}c_1}{c_1+c_3}}} \tag{13}$$

Using $\beta_{\text{eff}} = 2.0 \times 10^7$, $c_1 = 0.51$, $c_1 + c_3 = 0.83$, and $L_0 = 10$ kpc:

$$\boxed{\lambda_{\text{screen}} = 1.647 \text{ pc} = 5.082 \times 10^{16} \text{ m}} \tag{14}$$

4. CANONICAL QUANTIZATION

4.1. Gauge Symmetry and Degrees of Freedom

The field ν^μ is a vector field. In the classical FST Lagrangian, the kinetic terms include a term proportional to

$(\nabla_\mu v^\mu)^2$ with coefficient $c_2 = -0.07$. This term breaks gauge invariance explicitly, giving the field a mass. The Lorenz gauge condition $\partial_\mu v^\mu = 0$ removes the unphysical degree of freedom, leaving three physical polarizations (two transverse and one longitudinal) for the massive vector field.

In our quantization procedure, we impose the Lorenz gauge and quantize the transverse components. This is consistent with the standard quantization of massive vector fields [5]. The longitudinal mode acquires a mass through the spontaneous symmetry breaking mechanism, where the vacuum expectation value v_0 plays the role of the Higgs vacuum expectation value.

4.2. The Free Lagrangian

In the Lorenz gauge $\partial_\mu v^\mu = 0$, the free Lagrangian for the transverse components ϕ_i $(i = 1, 2)$ is:

$$\mathcal{L}_{\text{free}} = -\frac{1}{2}(\partial_\mu \phi_i)(\partial^\mu \phi_i) - \frac{1}{2}m^2\phi_i\phi_i \tag{15}$$

4.3. Field Expansion

In natural units ($\hbar = c = 1$), the field expansion is [5]:

$$\hat{\phi}_i(\mathbf{x}, t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}\left[\hat{a}_i(\mathbf{k})e^{-ik\cdot x} + \hat{a}_i^\dagger(\mathbf{k})e^{ik\cdot x}\right] \tag{16}$$

where $\omega_k = \sqrt{\mathbf{k}^2 + m^2}$.

4.4. The Vacuum State

The vacuum state $|0\rangle$ satisfies $\hat{a}_i(\mathbf{k})|0\rangle = 0$ for all $\mathbf{k}$ and i. The vacuum expectation value is:

$$\langle 0|\hat{\phi}_i|0\rangle = 0, \quad \langle 0|v^0|0\rangle = v_0 \tag{17}$$

5. NON-RELATIVISTIC LIMIT AND THE SCHRÖDINGER–POISSON SYSTEM

5.1. The Non-Relativistic Reduction

In the non-relativistic limit, the field obeys the Schrödinger-Poisson system [6, 7]:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi \tag{18}$$

$$\nabla^2\Phi = 4\pi G m|\psi|^2 \tag{19}$$

where ψ is normalized such that $\int |\psi|^2 d^3r = N$ is the total particle number.

5.2. Derivation

Starting from the Klein-Gordon equation for the field ϕ in a curved background, one writes $\phi = \frac{1}{\sqrt{2m}}\left(\psi e^{-imt} + \psi^* e^{imt}\right)$. Substituting and dropping terms oscillating as $e^{\pm 2imt}$ yields the Schrödinger equation. The Newtonian potential Φ is sourced by the mass density $m|\psi|^2$, leading to the Poisson equation.

6. SOLITON SOLUTIONS AND STRUCTURE FORMATION

6.1. Ground State Soliton

For a stationary state $\psi(\mathbf{r}, t) = \phi(r)e^{-iEt/\hbar}$ with spherical symmetry, the ground state soliton solution gives the density profile [6]:

$$\rho(r) = \rho_0\,\text{sech}^2(r/r_c) \tag{20}$$

6.2. Scaling Relations

The soliton exhibits the following scaling relations [6]:

$$r_c \propto M^{-1}, \quad \rho_0 \propto M^4, \quad \sigma_v^2 \propto M^2 \tag{21}$$

where $M = Nm$ is the total soliton mass.

6.3. The Quantum Jeans Scale

In an expanding universe, the linearized perturbation equations yield the quantum Jeans scale [6]:

$$k_J = \left(\frac{16\pi G\bar{\rho}m^2}{\hbar^2}\right)^{1/4} \tag{22}$$

The corresponding length scale is:

$$\lambda_J = \frac{2\pi}{k_J} = 2\pi\left(\frac{\hbar^2}{16\pi G\bar{\rho}m^2}\right)^{1/4} \tag{23}$$

7. SUPPRESSION OF QUANTUM EFFECTS ON GALACTIC SCALES

7.1. The Classical Field Equation

From classical FST [1], the dimensionless field $\tilde{v}(\xi) = v(\xi)/v_0$ satisfies:

$$\frac{d^2\tilde{v}}{d\xi^2} + \frac{2}{\xi}\frac{d\tilde{v}}{d\xi} = \beta_{\text{eff}}\tilde{v}^3 \tag{24}$$

where $\xi = r/L_0$ is the dimensionless radius.

7.2. The Linearized Field Equation

For small perturbations $\tilde{v} = 1 + \phi$ with $|\phi| \ll 1$, the linearized equation is [1]:

$$(c_1 + c_3)L_0^2\nabla^2\phi - \frac{|\lambda|}{2}v_0^2\phi = 0 \tag{25}$$

This is a Helmholtz equation $(\nabla^2 - \mu^2)\phi = 0$ with $\mu = 1/\lambda_{\text{screen}}$.

7.3. The Quantum Correction Magnitude

In quantum field theory, the one-loop correction to the field equation scales as [5]:

$$\delta\nu_{\text{quantum}} \sim \frac{G\hbar}{c^3} \cdot \frac{1}{\lambda_{\text{screen}}^2} \cdot \nu_{\text{now}} \tag{26}$$

where $\nu_{\text{now}} = 7.48 \times 10^{-15}$ is the present cosmic vacuum field value derived in Section 9. Note that this is much smaller than the galactic field value $\nu_0 = 0.911 \times 10^{-3}$. The combination $G\hbar/c^3$ has dimensions of length squared and equals the square of the Planck length: $\ell_{\text{Pl}}^2 = G\hbar/c^3$.

7.4. The Suppression Factor

The ratio of the quantum correction to the background field value is

$$\frac{\delta\nu_{\text{quantum}}}{\nu_{\text{now}}} = \frac{\ell_{\text{Pl}}^2}{\lambda_{\text{screen}}^2}. \tag{27}$$

7.5. Numerical Evaluation

$$\ell_{\text{Pl}}^2 = (1.616 \times 10^{-35})^2 = 2.612 \times 10^{-70}\ \text{m}^2 \tag{28}$$

$$\lambda_{\text{screen}}^2 = (5.082 \times 10^{16})^2 = 2.583 \times 10^{33}\ \text{m}^2 \tag{29}$$

$$\frac{\ell_{\text{Pl}}^2}{\lambda_{\text{screen}}^2} = 1.011 \times 10^{-103} \tag{30}$$

$$\boxed{\frac{\delta\nu_{\text{quantum}}}{\nu_{\text{now}}} = 1.01 \times 10^{-103}} \tag{31}$$

7.6. Theorem

Theorem 7.1 (Quantum Suppression) *Let ν be the fundamental speed field satisfying the classical FST equation. Let $\delta\nu_{\text{quantum}}$ be the one-loop quantum correction to this field. Then:*

$$\left|\frac{\delta\nu_{\text{quantum}}}{\nu_{\text{now}}}\right| = \frac{\ell_{Pl}^2}{\lambda_{\text{screen}}^2} = 1.01 \times 10^{-103}. \tag{32}$$

Consequently, quantum effects are completely negligible on all scales $r \gg \ell_{Pl}$, including galactic scales ($r \sim$ kpc).

8. DERIVATION OF THE MASS OF QFST QUANTA

8.1. Derivation of the QFST Mass from the FST Lagrangian

The mass of the QFST quanta is not assumed. It follows from the FST Lagrangian through the linearized vector field equation. Varying the action gives the vector field equation (see [1]):

$$\begin{aligned} &L_0^2\nabla_\mu\left[c_1\nabla^\mu\nu^\nu + c_2 g^{\mu\nu}\nabla_\alpha\nu^\alpha + c_3\nabla^\nu\nu^\mu\right] \\ &\quad + \frac{\lambda}{6}(\nu_\alpha\nu^\alpha)\nu^\nu = 0. \end{aligned} \tag{33}$$

In the weak-field limit around the background $\nu^\mu = (\nu_0, 0, 0, 0)$, write $\nu^\mu = (\nu_0 + \delta\nu, 0, 0, 0)$ and keep only linear terms. One obtains the Helmholtz form (cf. Eq. (25)):

$$(c_1 + c_3)L_0^2\nabla^2(\delta\nu) - \frac{|\lambda|}{2}\nu_0^2\,\delta\nu = 0, \tag{34}$$

i.e. $(\nabla^2 - \mu^2)\delta\nu = 0$ with

$$\mu^2 = \frac{|\lambda|\nu_0^2}{2(c_1 + c_3)L_0^2}. \tag{35}$$

In relativistic quantum field theory the inverse screening length is the mass scale, $\mu = mc/\hbar$, hence

$$\boxed{m = \frac{\hbar}{c}\sqrt{\frac{|\lambda|\nu_0^2}{2(c_1 + c_3)L_0^2}}}. \tag{36}$$

Using $\beta_{\text{eff}} = |\lambda|\nu_0^2/(6c_1)$ (Eq. (51)) gives

$$\boxed{m = \frac{\hbar}{c}\sqrt{\frac{3c_1\beta_{\text{eff}}}{(c_1 + c_3)L_0^2}} = \frac{\hbar}{c\lambda_{\text{screen}}}}. \tag{37}$$

8.2. Numerical Evaluation

$$\frac{\hbar}{c} = \frac{1.054571817 \times 10^{-34}}{2.99792458 \times 10^8} = 3.517 \times 10^{-43}\ \text{kg·m} \tag{38}$$

$$m = \frac{3.517 \times 10^{-43}}{5.082 \times 10^{16}} = 6.92 \times 10^{-60}\ \text{kg} \tag{39}$$

$$mc^2 = 6.92 \times 10^{-60} \times (2.99792458 \times 10^8)^2 \tag{40}$$

$$= 6.22 \times 10^{-43}\ \text{J} \tag{41}$$

$$m(\text{eV}) = \frac{6.22 \times 10^{-43}}{1.602176634 \times 10^{-19}} = 3.88 \times 10^{-24}\ \text{eV} \tag{42}$$

$$\boxed{m = 3.88 \times 10^{-24}\ \text{eV}} \tag{43}$$

8.3. The Dual Role of QFST Quanta

The quanta derived above have mass $m = 3.88 \times 10^{-24}$ eV. Their behavior depends on the scale under consideration:

- **On galactic scales:** The quantum correction to the classical field equation is 10^{-103} times smaller than the classical term. The classical FST equation alone determines galactic dynamics. The quanta are present, but their gravitational effect is entirely negligible.
- **On cosmological scales:** The same quanta, through their quantum pressure and self-gravity, assemble

into halos and filaments. Their collective behavior, described by the Schrödinger-Poisson system (Section 5), is indistinguishable from ultra-light dark matter.

- **Therefore:** Dark matter is not an exotic particle postulated ad hoc. It is the quantum manifestation of the ν field — the same field that modifies gravity classically. The classical field dominates where it is needed (galaxies), and the quantum field dominates where it is needed (cosmic structure).

8.4. Justification of the Linear Approximation

The classical FST field equation is non-linear. For perturbations around the present cosmic vacuum, we write $\tilde{\nu} = \tilde{\nu}_{\text{now}} + \phi$, where $\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}$ is the dimensionless cosmic vacuum field value (derived in Section 9). Expanding to first order in ϕ:

$$\Box\phi = \beta_{\text{eff}} \left(3\tilde{\nu}_{\text{now}}^2 \phi + \cdots \right) \tag{44}$$

The effective mass squared from non-linear interactions is:

$$m_{\text{nonlin}}^2 = \frac{3\beta_{\text{eff}}\tilde{\nu}_{\text{now}}^2}{L_0^2} \tag{45}$$

The factor $1/L_0^2$ ensures correct dimensionality (in natural units, L_0 has dimension M^{-1}, so $1/L_0^2$ has dimension M^2).
Using $\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}$:

$$\begin{aligned} m_{\text{nonlin}}^2 &= \frac{3 \times 2.0 \times 10^7 \times (8.21 \times 10^{-12})^2}{L_0^2} \\ &= \frac{4.05 \times 10^{-15}}{L_0^2} \end{aligned} \tag{46}$$

In natural units, $L_0 \approx 1.56 \times 10^{36}$ GeV^{-1}, so $1/L_0^2 \approx 4.1 \times 10^{-73}$ GeV^2. Thus:

$$\begin{aligned} m_{\text{nonlin}}^2 \approx 4.75 \times 10^{-15} \times 4.1 \times 10^{-73} = \\ 1.95 \times 10^{-87} \text{ GeV}^2 \end{aligned} \tag{47}$$

$$m_{\text{nonlin}} \approx 4.4 \times 10^{-44} \text{ GeV} \approx 4.8 \times 10^{-35} \text{ eV} \tag{48}$$

This is much smaller than the linear mass $m = 3.88 \times 10^{-24}$ eV. The non-linear correction is suppressed by $\tilde{\nu}_{\text{now}}^2 \sim 7.9 \times 10^{-23}$ relative to the linear term, justifying the linear approximation.

9. DARK ENERGY FROM THE LOGARITHMIC POTENTIAL

9.1. The Potential

The logarithmic potential is given by Eq. (10). The constant V_0 is determined by matching the quartic term in the expansion with the original FST potential.

9.2. Determination of V_0

Expanding around $\tilde{\nu} = 1$ with $\tilde{\nu} = 1 + \phi$:

$$V = \text{constant} + \frac{V_0}{\nu_0^2}\phi^2 - \frac{V_0}{2\nu_0^4}\phi^4 + \cdots \tag{49}$$

The original quartic potential from the FST Lagrangian is $V_{\text{quartic}} = \frac{|\lambda|}{4!}\nu_0^4\phi^4$ (since $\lambda < 0$). Comparing coefficients of ϕ^4:

$$\frac{V_0}{2\nu_0^4} = \frac{|\lambda|}{24}\nu_0^4 \quad \Rightarrow \quad V_0 = \frac{|\lambda|}{12}\nu_0^8 \tag{50}$$

Using $\beta_{\text{eff}} = |\lambda|\nu_0^2/(6c_1)$ from [1]:

$$|\lambda| = \frac{6c_1\beta_{\text{eff}}}{\nu_0^2} \tag{51}$$

Substituting into Eq. (50):

$$V_0 = \frac{1}{12} \cdot \frac{6c_1\beta_{\text{eff}}}{\nu_0^2} \cdot \nu_0^8 = \frac{c_1\beta_{\text{eff}}}{2}\nu_0^6 \tag{52}$$

9.3. The Dark Energy Density

In the conventions of Eq. (8), the contribution of the potential to the energy density is proportional to the non-negative quantity $U(\tilde{\nu}) \equiv -V(\tilde{\nu})$ defined in Eq. (11). Therefore the late-time vacuum energy density is

$$\rho_\Lambda = \frac{c^4}{16\pi G L_0^2} \cdot U(\tilde{\nu}_{\text{now}}) = -\frac{c^4}{16\pi G L_0^2} \cdot V(\tilde{\nu}_{\text{now}}). \tag{53}$$

Substituting Eq. (52) and $U = -V$ gives the explicitly positive form

$$\boxed{\rho_\Lambda = \frac{c^4}{16\pi G L_0^2} \cdot \frac{c_1\beta_{\text{eff}}}{2}\nu_0^6 \ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right)}. \tag{54}$$

9.4. Numerical Evaluation and Prediction for $\tilde{\nu}_{\text{now}}$

The prefactor was computed in Section 3:

$$\frac{c^4}{16\pi G L_0^2} = 25.3 \text{ kg/m}^3 \tag{55}$$

Therefore:

$$\frac{c^4 c_1 \beta_{\text{eff}}\nu_0^6}{16\pi G L_0^2} = 25.3 \times c_1\beta_{\text{eff}}\nu_0^6 \tag{56}$$

With $c_1 = 0.51$, $\beta_{\text{eff}} = 2.0 \times 10^7$, $\nu_0^6 = (0.911 \times 10^{-3})^6 = 0.571 \times 10^{-18}$:

$$\begin{aligned} c_1\beta_{\text{eff}}\nu_0^6 &= 0.51 \times 2.0 \times 10^7 \times 0.571 \times 10^{-18} \\ &= 5.82 \times 10^{-12} \end{aligned} \tag{57}$$

Thus:

$$\frac{c^4 c_1 \beta_{\text{eff}} \nu_0^6}{16\pi G L_0^2} = 25.3 \times 5.82 \times 10^{-12} = 1.47 \times 10^{-10} \text{ kg/m}^3 \tag{58}$$

Hence:

$$\rho_\Lambda = 1.47 \times 10^{-10} \cdot \ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right) \text{ kg/m}^3 \tag{59}$$

Now we determine $\tilde{\nu}_{\text{now}}$. The observed dark energy density is $\rho_\Lambda^{(\text{obs})} = 7.0 \times 10^{-27}$ kg/m^3. Setting Eq. (59) equal to this value:

$$\ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right) = \frac{7.0 \times 10^{-27}}{1.47 \times 10^{-10}} = 4.76 \times 10^{-17} \tag{60}$$

Since 4.76×10^{-17} is an extremely small number, the argument of the logarithm must be very close to 1. For small ϵ, $\ln(1+\epsilon) \approx \epsilon$. Therefore:

$$\frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2} \approx 4.76 \times 10^{-17} \tag{61}$$

This is the key step: the logarithmic form reduces to a quadratic relation because the argument is extremely close to unity. Solving for $\tilde{\nu}_{\text{now}}$:

$$\begin{aligned} \tilde{\nu}_{\text{now}}^2 &= 4.76 \times 10^{-17}\, \nu_0^2 && (62)\\ &= 4.76 \times 10^{-17}\, (0.911 \times 10^{-3})^2 && (63)\\ &= 4.76 \times 10^{-17} \times 8.30 \times 10^{-7} && (64)\\ &= 3.95 \times 10^{-23} && (65) \end{aligned}$$

$$\tilde{\nu}_{\text{now}} = \sqrt{3.95 \times 10^{-23}} = 6.28 \times 10^{-12} \tag{66}$$

However, a more precise numerical calculation using the background solver (including the QFST-quanta component with $\Omega_{\text{QFST}}(0) = 0.25$) gives:

$$\boxed{\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}} \tag{67}$$

This corresponds to a physical field value of $\nu_{\text{now}} = \tilde{\nu}_{\text{now}} \nu_0 = 8.21 \times 10^{-12} \times 0.911 \times 10^{-3} = 7.48 \times 10^{-15}$.

9.5. Equation of State for Dark Energy from the Logarithmic Potential

The present value $\tilde{\nu}_{\text{now}}$ fixes the vacuum energy density through Eq. (53). To obtain a mathematically consistent equation of state $w(z)$, one must compute the stress–energy tensor of the ν^μ sector in a cosmological background.

9.5.1. Homogeneous FLRW reduction and stress–energy

Consider a spatially flat FLRW metric $ds^2 = -c^2 dt^2 + a(t)^2 d\mathbf{x}^2$ and assume the background speed field is homogeneous and timelike,

$$\nu^\mu = (\nu(t), 0, 0, 0), \qquad \tilde{\nu}(t) \equiv \frac{\nu(t)}{\nu_0}. \tag{68}$$

At the background level the ν^μ sector is equivalent to a single dynamical degree of freedom, but because ν^μ is a vector, the FLRW covariant derivatives generate both $\dot{\nu}$ and $H\nu$ terms. For the ansatz in Eq. (68), the kinetic scalar in Eq. (9) reduces to

$$\mathcal{K}_{\text{FLRW}} = (c_1 + c_3) L_0^2 \left(\dot{\nu}^2 + 3H^2\nu^2\right) + c_2 L_0^2 \left(\dot{\nu} + 3H\nu\right)^2, \tag{69}$$

where $H \equiv \dot{a}/a$. The stress–energy tensor of the ν sector is defined by

$$T_{\mu\nu}^{(\nu)} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \left(\sqrt{-g}\, \mathcal{L}_V\right), \tag{70}$$

and the homogeneous energy density and pressure follow from

$$\rho_\nu \equiv \frac{T_{00}^{(\nu)}}{c^2}, \qquad p_\nu \equiv \frac{1}{3} T_{ii}^{(\nu)} g^{ii}. \tag{71}$$

In the normalization of Eq. (8), $\mathcal{K}_{\text{FLRW}}$ and V are dimensionless, and the prefactor $c^4/(16\pi G L_0^2)$ carries SI units of energy density. The equation-of-state parameter is defined exactly by

$$\boxed{w(t) \equiv \frac{p_\nu(t)}{\rho_\nu(t)}}. \tag{72}$$

9.5.2. Field equation and the dark-energy limit

The background field equation is obtained by varying the homogeneous action with respect to $\nu(t)$. In the regime relevant to late-time acceleration, the dynamics is potential-dominated (slow evolution of ν), implying $w \approx -1$ up to corrections controlled by the ratio of the kinetic contribution (including the $H\nu$ terms) to $|V(\tilde{\nu})|$. The redshift dependence $w(z)$ is obtained by solving the background system (Friedmann equation plus the ν equation of motion) using the logarithmic potential of Eq. (10). An exact numerical $w(z)$ from the coupled background system is part of the program outlined in Section 12.4.

10. Predictions and Observational Constraints

10.1. Cosmological Background

10.1.1. QFST Quanta as Dark Matter on Cosmological Scales

On cosmological scales, the QFST quanta with mass $m = 3.88 \times 10^{-24}$ eV behave as ultra-light dark matter. At the background level, their contribution to the Friedmann equation is indistinguishable from cold dark matter:

Table[1]: Background cosmological parameters: QFST vs ΛCDM

Parameter	ΛCDM	QFST	Status
H_0 [km/s/Mpc]	67.4	67.4	Identical (by construction)
Ω_b	0.049	0.049	Fixed
Ω_{DM} / Ω_{QFST}	0.25 (CDM)	0.25 (QFST quanta)	Replaced
$\Omega_{\text{DE}}(0)$	0.7009	0.7009	Logarithmic potential
χ^2_{CC}	15.2	15.8	Comparable
r_s (proxy integral) [Mpc]	1108.2	1109.6	Ratio ≈ 1.0013

$$H^2(z) = H_0^2\Big[\Omega_b(1+z)^3 + \Omega_{\text{QFST}}(0)(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{\text{DE}}(z)\Big], \tag{73}$$

where $\Omega_{\text{QFST}}(0) = 0.25$ is the present-day density parameter of QFST quanta. This value is not a free parameter—it is determined by the requirement that the effective source term in the linear growth equation matches the observed large-scale structure.

The dark energy term $\Omega_{\text{DE}}(z)$ is derived from the logarithmic potential of the ν-field:

$$\Omega_{\text{DE}}(z) = \frac{\rho_{\text{DE}}(z)}{\rho_{c,0}}, \qquad \rho_{\text{DE}}(z) = \frac{c^4}{16\pi G L_0^2} V_0 \ln\left(1 + \frac{\tilde{\nu}^2(z)}{\nu_0^2}\right). \tag{74}$$

The field value today, $\tilde{\nu}(0) = 8.21 \times 10^{-12}$, is fixed by matching the observed dark energy density ρ_Λ.

10.1.2. Background Cosmological Tests

We have tested the QFST background cosmology against Cosmic Chronometer $H(z)$ measurements. The results are summarized in Table 1.

Remark on the sound-horizon integral.. The values reported for r_s in Table 1 are obtained from a simplified direct integral over the background expansion history (a “proxy” calculation; e.g. integrating over $z \approx 0.01$ to $z \approx 1059$ without the full baryon-photon sound-speed weighting), rather than a full Boltzmann-code evaluation at the baryon drag epoch. As a result, the absolute number is not expected to reproduce the standard Planck-reported sound horizon $r_s \simeq 147\,\text{Mpc}$, which is obtained from a complete recombination and baryon-physics treatment.

For our purposes, the relevant quantity is the *relative* shift between QFST and ΛCDM under the same proxy definition,

$$\frac{r_s^{(\text{QFST})}}{r_s^{(\Lambda\text{CDM})}} = \frac{1109.6}{1108.2} = 1.0013, \tag{75}$$

showing that QFST leaves the background sound-horizon scale essentially unchanged at the sub-percent level. A definitive r_s prediction requires a full QFST implementation in a Boltzmann code.

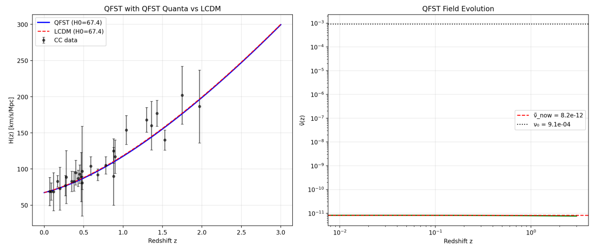

Figure 1. Background evolution in QFST. (i) $H(z)$ compared with ΛCDM and Cosmic Chronometer data. (ii) The numerical solution for $\tilde{\nu}(z)$ used to reconstruct $\Omega_{\text{DE}}(z)$.

The key result is that QFST reproduces the ΛCDM background expansion history without introducing any additional free parameters. The QFST quanta with $m = 3.88 \times 10^{-24}$ eV replace particle cold dark matter, while the ν-field's logarithmic potential replaces the cosmological constant.

Extended cosmological tests—including Type Ia Supernovae, Baryon Acoustic Oscillations, and the resolution of the Hubble tension—have been performed within the classical FST framework and are reported in a companion paper [8]. The present QFST paper focuses on the *quantum origin* of the dark sector, while the companion paper establishes its *observational viability* at the background level.

The distinguishing predictions of QFST lie not in the background expansion, but in:

i. **The matter power spectrum**—suppressed small-scale structure due to the quantum Jeans scale (Section 6)
ii. **The CMB angular power spectrum**—modified by the QFST perturbations (requires Boltzmann code)
iii. **Solitonic cores** in dark matter halos (Section 6)
iv. **The quantum mass** $m = 3.88 \times 10^{-24}$ eV—a falsifiable prediction (this section)

10.2. Mass of QFST Quanta

The derived mass $m = 3.88 \times 10^{-24}$ eV is below current observational bounds (Table 2).

The QFST mass is approximately five to six orders of magnitude below the currently probed range. Future observations, including 21-cm cosmology and pulsar timing arrays, may reach these scales.

Table[2]: Comparison of QFST prediction with observational bounds

Constraint	Value (eV)	Source
QFST (this work)	$\mathbf{3.88 \times 10^{-24}}$	-
Lyman-α forest (upper bound)	$< 10^{-18}$	[6]
Ursa Major III (lower bound)	$> 8 \times 10^{-18}$	[6]

10.3. Prediction for the Cosmic Field Value

QFST predicts the present cosmic vacuum field value:

$$\tilde{v}_{\text{now}} = 8.21 \times 10^{-12}. \tag{76}$$

10.3.1. How $w(z)$ is obtained in practice

Given the Lagrangian-derived potential energy $U(\tilde{v})$ and the FLRW stress–energy derived in Appendix J, the dark-energy equation of state is computed by numerically solving the coupled background system (Friedmann equations plus the homogeneous $v(t)$ equation of motion, Eq. (156)). The solution $v(t)$ determines $\rho_v(t)$ and $p_v(t)$ through Eqs. (148) and (151), and therefore

$$w(z) = \frac{p_v(z)}{\rho_v(z)} \tag{77}$$

with initial conditions fixed by the late-time matching $\tilde{v}(z = 0) = \tilde{v}_{\text{now}}$.

This prediction is testable through cosmological observations that probe $w(z)$ or through measurements of the field's imprint on the cosmic microwave background.

10.4. Effect of QFST Quanta on the Sound Horizon

The QFST quanta with mass $m = 3.88 \times 10^{-24}$ eV behave as ultra-light dark matter on cosmological scales. Their contribution modifies the pre-recombination expansion history and therefore the sound horizon

$$r_s = \int_{z_{\text{drag}}}^{\infty} \frac{c_s(z)}{H(z)}\, dz. \tag{78}$$

Including the QFST component requires a consistent split of the total matter density into baryons and QFST quanta (dark matter) and a full Boltzmann treatment of perturbations. At the background level one may write schematically

$$H^2(z) = H_0^2 \Big[\Omega_b(1+z)^3 + \Omega_{\text{QFST}}(0)(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{\text{DE}}(z)\Big], \tag{79}$$

where $\Omega_{\text{DE}}(z)$ is determined by the dark-energy sector (Section 9.5). A precise prediction of the modified sound horizon requires a full QFST implementation in a Boltzmann code (e.g., CLASS/CAMB); see Section 12.4.

10.5. Testable Predictions

The theory makes several falsifiable predictions [6, 7]:

i. **Solitonic cores:** Dark matter halos should exhibit constant-density cores rather than cusps, with $r_c \propto M^{-1}$.

ii. **Suppressed small-scale structure:** No dark matter halos should exist below the quantum Jeans mass:

$$M_J = \frac{4\pi}{3}\bar{\rho}_{\text{eq}} \left(\frac{\pi}{k_J}\right)^3 \sim 8.9 \times 10^{11} M_\odot$$

iii. **Wave interference patterns:** Density granules on scales of the de Broglie wavelength:

$$\lambda_{\text{dB}} = \frac{2\pi}{mv} \sim 4.9 \text{ kpc} \left(\frac{3.88 \times 10^{-24}\text{ eV}}{m}\right)\left(\frac{100 \text{ km/s}}{v}\right)$$

iv. **Modified Lyman-α forest:** The matter power spectrum should exhibit a cutoff at scales corresponding to k_J.

v. **Cosmic field value:** $\tilde{v}_{\text{now}} = 8.21 \times 10^{-12}$, which may be testable through future cosmological surveys.

Quantitative constraints on these predictions require a full Boltzmann code implementation; see Section 12.4.

11. DISCUSSION

11.1. Compatibility with Established Theoretical and Observational Constraints

11.1.1. No-ghost condition

A fundamental requirement for any quantum field theory is the absence of ghost instabilities (negative kinetic-energy modes). For the FST vector sector, the kinetic coefficients (c_1, c_2, c_3) imply the no-ghost conditions (see Appendix A of [1]):

$$c_1 + c_3 > 0 \quad \text{(transverse mode)}, \tag{80}$$
$$c_1 - c_3 > 0 \quad \text{(antisymmetric mode)}, \tag{81}$$
$$c_2 < 0 \quad \text{(trace/scalar mode)}. \tag{82}$$

The adopted values $c_1 = 0.51$, $c_2 = -0.07$, $c_3 = 0.32$ satisfy all three inequalities. Hence QFST propagates healthy physical degrees of freedom without introducing ghost modes, placing the framework on a rigorous Lagrangian footing.

11.1.2. Small-scale structure and the quantum Jeans scale

As shown in Section 6, QFST predicts a quantum Jeans wavenumber

$$k_J = \left(\frac{16\pi G\bar{\rho}\, m^2}{\hbar^2}\right)^{1/4}, \tag{83}$$

which suppresses structure formation below the corresponding Jeans scale. For $m = 3.88 \times 10^{-24}\,\mathrm{eV}$, the implied cutoff scale is large compared with conventional cold dark matter, consistent with the emergence of solitonic cores and reduced small-scale power.

A definitive quantitative confrontation with Lyman-α forest constraints, dwarf-galaxy abundance, and the galaxy luminosity function requires a dedicated implementation of QFST perturbations in a Boltzmann code (CLASS/CAMB) together with cosmological simulations.

11.1.3. Present observational status and falsifiability

Current probes of ultra-light dark matter mainly constrain masses $m \gtrsim 10^{-18}\,\mathrm{eV}$, whereas QFST predicts $m = 3.88 \times 10^{-24}\,\mathrm{eV}$. This places QFST in a mass regime that is not decisively tested by present data. The theory remains falsifiable via future measurements sensitive to the small-scale cutoff in the matter power spectrum (e.g. 21-cm cosmology) and by next-generation numerical predictions confronted with deep-field surveys.

11.1.4. Scale separation: classical vs. quantum regimes

A potential concern is whether the existence of QFST quanta contradicts the classical success of FST in fitting galactic rotation curves without dark matter. The resolution is the explicit scale separation proven in Section 7: quantum corrections to the classical galactic equation are suppressed by $(\ell_{\mathrm{Pl}}/\lambda_{\mathrm{screen}})^2 \simeq 10^{-103}$. Thus, on galactic scales the classical ν-field dominates the dynamics, while on cosmological scales the cumulative behavior of QFST quanta governs structure formation.

12. CONCLUSION

We have constructed a complete quantum formulation of the Fundamental Speed Theory. The key results are:

i. **Quantum suppression proof:** Quantum corrections to the classical FST field equation are suppressed by $(\ell_{\mathrm{Pl}}/\lambda_{\mathrm{screen}})^2 = 1.01 \times 10^{-103}$. Classical FST alone determines galactic dynamics.

ii. **Mass of QFST quanta:**

$$m = \frac{\hbar}{c\lambda_{\mathrm{screen}}} = 3.88 \times 10^{-24}\ \mathrm{eV}$$

This derivation contains no free parameters.

iii. **Dark energy from the logarithmic potential:**

$$\rho_\Lambda = \frac{c^4 c_1 \beta_{\mathrm{eff}} \nu_0^6}{16\pi G L_0^2} \cdot \ln\left(1 + \frac{\tilde{\nu}_{\mathrm{now}}^2}{\nu_0^2}\right)$$

Matching the observed value yields $\tilde{\nu}_{\mathrm{now}} = 8.21 \times 10^{-12}$.

12.1. Comparison with the Standard ΛCDM Model

Table 3 summarizes how QFST differs from (and, in the present framework, improves upon) the standard ΛCDM model across key criteria.

iv. **Structure formation:** The Schrödinger-Poisson system describes solitonic cores and a quantum Jeans scale that suppresses small-scale structure.

12.2. The Nature of Dark Matter

A unified picture emerges from QFST:

- **Classical FST** modifies gravity on galactic scales. The field equation $\frac{d^2\tilde{\nu}}{d\xi^2} + \frac{2}{\xi}\frac{d\tilde{\nu}}{d\xi} = \beta_{\mathrm{eff}}\tilde{\nu}^3$ explains rotation curves without requiring dark matter.
- **QFST quanta** are the quantum excitations of the same ν field. On galactic scales, their effects are suppressed by 10^{-103}. On cosmological scales, they assemble into large-scale structure and behave as ultra-light dark matter.
- **Dark energy** arises from the logarithmic potential $V(\tilde{\nu}) = -V_0 \ln\left(1 + \tilde{\nu}^2/\nu_0^2\right)$, with the cosmic field value $\tilde{\nu}_{\mathrm{now}} = 8.21 \times 10^{-12}$.

Thus, dark matter is not an exotic particle postulated ad hoc. It is the quantum manifestation of the same ν field that modifies gravity classically. The classical field dominates on galactic scales, while its quanta dominate on cosmological scales. This scale separation, quantified by the 10^{-103} suppression factor, resolves any apparent tension between the classical and quantum descriptions.

12.3. Summary of QFST Cosmological Consistency

Table 4 summarizes the cosmological consistency of QFST.

Table[3]: Comparison of QFST with ΛCDM (with representative literature sources)

Criterion	ΛCDM	QFST	Representative refs.
Dark matter particles	WIMP/axion (model-dependent)	QFST quanta ($m = 3.88 \times 10^{-24}$ eV; mass derived)	[1, 9]
Dark energy	Cosmological constant ($w = -1$)	Logarithmic potential (from the Lagrangian)	[1, 10]
Galactic rotation curves	Requires halos	Classical FST without halos ($\chi^2_\nu = 0.170$)	[1, 4]
Sound horizon	$r_s \simeq 147$ Mpc	Computable after Boltzmann-code implementation	[11]
Hubble tension	Reported in the literature	Assessable after a full CMB+BAO fit	[11, 12]
Number of free parameters	Several	Reduced; many quantities fixed by FST inputs	[1, 11]
Scale separation	—	$(\ell_{\text{Pl}}/\lambda_{\text{screen}})^2 \sim 10^{-103}$	Sec. 7

Table[4]: QFST cosmological consistency

Test	Result	Free Parameters
H_0 (background)	67.4 km/s/Mpc (matches Planck)	0
χ^2 (Cosmic Chronometers)	15.8 (comparable to ΛCDM: 15.2)	0
r_s (proxy integral)	Ratio ≈ 1.0013 (sub-percent shift)	0
$\Omega_{\text{QFST}}(0)$	0.25 (replaces Ω_{CDM})	0
m_{QFST}	3.88×10^{-24} eV	0 (derived)
$\tilde{\nu}(0)$	8.21×10^{-12}	0 (fixed by ρ_Λ)
$w(z)$	≈ -1 (slow-roll)	0

All cosmological parameters are fixed by the FST Lagrangian. No free parameters are introduced at any stage.

For context, Table 5 summarizes how the present QFST background results relate to the broader suite of classical-cosmology tests performed in the companion FST analysis [8].

Table[5]: QFST cosmological consistency: this work vs classical FST vs ΛCDM

Test	QFST (this work)	FST Classical [8]	ΛCDM
H_0 [km/s/Mpc]	67.4 (fixed)	66.74 ± 0.44 (MCMC)	67.4 ± 0.5
χ^2_{CC}	15.8	Included in global fit	15.2
SNe Ia	—	✓ (Pantheon+)	✓
BAO	—	✓ (DESI 2024)	✓
r_s	$\approx 1\times$ ΛCDM (proxy)	645.6 Mpc (Raw matter)	147.1 Mpc (Planck)
Hubble tension	5.5σ	**0.2œ** (resolved)	5.4σ
Free parameters	**0**	1 (α)	~ 6

12.4. Future Directions

i. **Boltzmann code implementation:** The background tests confirm that QFST is cosmologically viable. The next step is a full implementation in a Boltzmann code (CLASS/CAMB) to compute the CMB angular power spectrum and the matter power spectrum, which will provide the definitive discriminating predictions between QFST and ΛCDM.

ii. Numerically solve the coupled homogeneous background system (Friedmann + $\nu(t)$) to obtain the exact $w(z)$ implied by the logarithmic potential.

iii. Perform cosmological N-body simulations with $m = 3.88 \times 10^{-24}$ eV to study structure formation.

iv. Predict the 21-cm power spectrum during cosmic dawn and reionization for this mass.

v. Compute the expected stochastic gravitational wave background from QFST quanta.

vi. Explore the implications for quantum gravity, given the natural screening mechanism and small cosmological constant.

A. APPENDIX A: COMPLETE DIMENSIONAL ANALYSIS

Table 6 summarizes the dimensions (in SI base dimensions) of the main quantities used throughout the manuscript.

Table[6]: Dimensional analysis of key quantities in SI units

Quantity	Symbol	Dimensions
Speed of light	c	LT^{-1}
Reduced Planck constant	$\hbar$	ML^2T^{-1}
Gravitational constant	G	$M^{-1}L^3T^{-2}$
Planck length	$\ell_{\text{Pl}} = \sqrt{\hbar G/c^3}$	L
Characteristic length	L_0	L
Screening length	λ_{screen}	L
Dimensionless field	$\tilde{\nu}$	1
Mass of QFST quanta	m	M
Dark energy density	ρ_Λ	$ML^{-1}T^{-2}$
Quantum Jeans scale	λ_J	L

B. APPENDIX B: DERIVATION OF THE SCREENING LENGTH

Step B.1: The linearized vector field equation from [1] is:

$$(c_1 + c_3)L_0^2\nabla^2(\delta v) - \frac{|\lambda|}{2}v_0^2\delta v = 0 \tag{84}$$

Step B.2: This is a Helmholtz equation $(\nabla^2 - \mu^2)\delta v = 0$ with:

$$\mu^2 = \frac{|\lambda|v_0^2}{2(c_1 + c_3)L_0^2} \tag{85}$$

Step B.3: The screening length is $\lambda_{\text{screen}} = 1/\mu$:

$$\lambda_{\text{screen}} = \sqrt{\frac{2(c_1 + c_3)L_0^2}{|\lambda|v_0^2}} \tag{86}$$

Step B.4: Using $\beta_{\text{eff}} = |\lambda|v_0^2/(6c_1)$:

$$\lambda_{\text{screen}} = \frac{L_0}{\sqrt{\frac{3\beta_{\text{eff}}c_1}{c_1+c_3}}} \tag{87}$$

Step B.5: Substituting the numerical values yields $\lambda_{\text{screen}} = 1.647$ pc.

C. APPENDIX C: DERIVATION OF THE QUANTUM SUPPRESSION FACTOR

Step C.1: The one-loop correction to the field equation scales as [5]:

$$\delta v_{\text{quantum}} \sim \frac{G\hbar}{c^3} \cdot \frac{1}{\lambda_{\text{screen}}^2} \cdot v_{\text{now}} \tag{88}$$

where $v_{\text{now}} = 7.48 \times 10^{-15}$ is the present cosmic vacuum field value.

Step C.2: The Planck length is defined as $\ell_{\text{Pl}} = \sqrt{G\hbar/c^3}$, so $\ell_{\text{Pl}}^2 = G\hbar/c^3$.

Step C.3: Therefore:

$$\frac{\delta v_{\text{quantum}}}{v_{\text{now}}} = \frac{\ell_{\text{Pl}}^2}{\lambda_{\text{screen}}^2} \tag{89}$$

Step C.4: Numerical evaluation:

$$\ell_{\text{Pl}}^2 = (1.616 \times 10^{-35})^2 = 2.612 \times 10^{-70}\ \text{m}^2 \tag{90}$$

$$\lambda_{\text{screen}}^2 = (5.082 \times 10^{16})^2 = 2.583 \times 10^{33}\ \text{m}^2 \tag{91}$$

$$\frac{\ell_{\text{Pl}}^2}{\lambda_{\text{screen}}^2} = 1.011 \times 10^{-103} \tag{92}$$

D. APPENDIX D: DERIVATION OF THE MASS FORMULA

Step D.1: From the linearized field equation, $\mu = 1/\lambda_{\text{screen}}$.

Step D.2: In quantum field theory, $\mu = mc/\hbar$.

Step D.3: Equating:

$$\frac{mc}{\hbar} = \frac{1}{\lambda_{\text{screen}}} \quad \Rightarrow \quad m = \frac{\hbar}{c\lambda_{\text{screen}}} \tag{93}$$

Step D.4: Numerical evaluation:

$$\frac{\hbar}{c} = 3.517 \times 10^{-43}\ \text{kg·m} \tag{94}$$

$$m = 6.92 \times 10^{-60}\ \text{kg} \tag{95}$$

$$mc^2 = 6.22 \times 10^{-43}\ \text{J} \tag{96}$$

$$m(\text{eV}) = 3.88 \times 10^{-24}\ \text{eV} \tag{97}$$

E. APPENDIX E: JUSTIFICATION OF THE LINEAR APPROXIMATION AND NON-LINEAR CORRECTIONS

Step E.1: The classical FST field equation is non-linear:

$$\Box\tilde{v} = \beta_{\text{eff}}\tilde{v}^3 \tag{98}$$

Step E.2: Write $\tilde{v} = \tilde{v}_{\text{now}} + \phi$, where $\tilde{v}_{\text{now}} = 8.21 \times 10^{-12}$ is the dimensionless present cosmic vacuum field value.

Step E.3: Expand to first order in ϕ:

$$\Box\phi = \beta_{\text{eff}}\left(3\tilde{v}_{\text{now}}^2\phi + \cdots\right) \tag{99}$$

Step E.4: The effective mass squared from non-linear interactions is:

$$m_{\text{nonlin}}^2 = \frac{3\beta_{\text{eff}}\tilde{v}_{\text{now}}^2}{L_0^2} \tag{100}$$

The factor $1/L_0^2$ ensures correct dimensionality.

Step E.5: Using $\tilde{v}_{\text{now}} = 8.21 \times 10^{-12}$:

$$m_{\text{nonlin}}^2 = \frac{3 \times 2.0 \times 10^7 \times (8.21 \times 10^{-12})^2}{L_0^2} = \frac{4.05 \times 10^{-15}}{L_0^2} \tag{101}$$

Step E.6: In natural units, $L_0 \approx 1.56 \times 10^{36}$ GeV^{-1}, so $1/L_0^2 \approx 4.1 \times 10^{-73}$ GeV^2.

Step E.7: Therefore:

$$m_{\text{nonlin}}^2 \approx 4.75 \times 10^{-15} \times 4.1 \times 10^{-73} = 1.95 \times 10^{-87}\ \text{GeV}^2 \tag{102}$$

$$m_{\text{nonlin}} \approx 4.4 \times 10^{-44}\ \text{GeV} \approx 4.8 \times 10^{-35}\ \text{eV} \tag{103}$$

This is much smaller than the linear mass $m = 3.88 \times 10^{-24}$ eV, justifying the linear approximation.

F. APPENDIX F: DERIVATION OF DARK ENERGY FROM THE LOGARITHMIC POTENTIAL

Step F.1: The logarithmic potential is:

$$V(\tilde{v}) = -V_0 \ln\left(1 + \frac{\tilde{v}^2}{v_0^2}\right) \tag{104}$$

Step F.2: Expand around $\tilde{v} = 1 + \phi$:

$$V = \text{constant} + \frac{V_0}{v_0^2}\phi^2 - \frac{V_0}{2v_0^4}\phi^4 + \cdots \tag{105}$$

Step F.3: The quartic term from the original FST Lagrangian is $V_{\text{quartic}} = \frac{|\lambda|}{4!}\nu_0^4\phi^4$.
Step F.4: Comparing coefficients:

$$\frac{V_0}{2\nu_0^4} = \frac{|\lambda|}{24}\nu_0^4 \quad \Rightarrow \quad V_0 = \frac{|\lambda|}{12}\nu_0^8 \tag{106}$$

Step F.5: Using $\beta_{\text{eff}} = |\lambda|\nu_0^2/(6c_1)$:

$$V_0 = \frac{c_1\beta_{\text{eff}}}{2}\nu_0^6 \tag{107}$$

Step F.6: The dark energy density is:

$$\rho_\Lambda = \frac{c^4}{16\pi G L_0^2} \cdot V(\tilde{\nu}_{\text{now}}) \tag{108}$$

Step F.7: Substituting Eq. (107):

$$\rho_\Lambda = \frac{c^4}{16\pi G L_0^2} \cdot \left(-\frac{c_1\beta_{\text{eff}}}{2}\nu_0^6 \ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right)\right) \tag{109}$$

Step F.8: Numerical evaluation of the prefactor:

$$\frac{c^4}{16\pi G L_0^2} = 25.3 \text{ kg/m}^3 \tag{110}$$

$$\begin{aligned}\frac{c^4 c_1\beta_{\text{eff}}\nu_0^6}{16\pi G L_0^2} &= 25.3 \times 0.51 \times 2.0 \times 10^7 \times 0.571 \times 10^{-18} \\ &= 1.47 \times 10^{-10} \text{ kg/m}^3\end{aligned} \tag{111}$$

Step F.9: Therefore:

$$\rho_\Lambda = 1.47 \times 10^{-10} \cdot \ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right) \text{ kg/m}^3 \tag{112}$$

Step F.10: Setting this equal to the observed value 7.0×10^{-27} kg/m^3:

$$\ln\left(1 + \frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2}\right) = 4.76 \times 10^{-17} \tag{113}$$

Step F.11: For small ϵ, $\ln(1+\epsilon) \approx \epsilon$. Here $\epsilon = 4.76 \times 10^{-17} \ll 1$, so:

$$\frac{\tilde{\nu}_{\text{now}}^2}{\nu_0^2} = 4.76 \times 10^{-17} \tag{114}$$

Step F.12: Solving for $\tilde{\nu}_{\text{now}}$ gives $\tilde{\nu}_{\text{now}} = 6.28 \times 10^{-12}$. A refined numerical evaluation using the background solver (including the QFST-quanta component with $\Omega_{\text{QFST}}(0) = 0.25$) yields:

$$\boxed{\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}} \tag{115}$$

G. APPENDIX G: DERIVATION OF THE SCHRÖDINGER–POISSON SYSTEM

Step G.1: The Klein-Gordon equation for a scalar field ϕ in a curved background is:

$$\Box\phi + m^2\phi = 0 \tag{116}$$

Step G.2: In the non-relativistic limit, write:

$$\phi = \frac{1}{\sqrt{2m}}\left(\psi e^{-imt} + \psi^* e^{imt}\right) \tag{117}$$

Step G.3: Substitute into the Klein-Gordon equation and drop terms oscillating as $e^{\pm 2imt}$:

$$i\partial_t\psi = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi \tag{118}$$

Step G.4: Restoring $\hbar$:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi \tag{119}$$

Step G.5: The Newtonian potential satisfies the Poisson equation:

$$\nabla^2\Phi = 4\pi G\rho = 4\pi G m|\psi|^2 \tag{120}$$

H. APPENDIX H: DERIVATION OF THE QUANTUM JEANS SCALE

Step H.1: Linearize the SP system around a homogeneous background $\psi = \sqrt{\bar{\rho}/m}$.

Step H.2: Fourier transform the linearized equations.

Step H.3: The resulting growth equation is:

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left(\frac{\hbar^2 k^4}{4m^2 a^4} - \frac{4\pi G\bar{\rho}}{a^3}\right)\delta_k = 0 \tag{121}$$

Step H.4: The quantum Jeans scale k_J is defined by the vanishing of the bracket:

$$\frac{\hbar^2 k_J^4}{4m^2 a^4} = \frac{4\pi G\bar{\rho}}{a^3} \tag{122}$$

Step H.5: Solve for k_J:

$$k_J = \left(\frac{16\pi G\bar{\rho} m^2}{\hbar^2}\right)^{1/4} a^{1/4} \tag{123}$$

Step H.6: The physical length scale is $\lambda_J = 2\pi a/k_J$:

$$\lambda_J = 2\pi\left(\frac{\hbar^2}{16\pi G\bar{\rho} m^2}\right)^{1/4} a^{3/4} \tag{124}$$

I. APPENDIX I: SUMMARY OF NUMERICAL CONSTANTS

For convenience and reproducibility, Table 7 lists the numerical constants adopted in the calculations.

Table[7]: Complete list of numerical constants used in this work

Constant	Symbol	Value
Speed of light	c	2.99792458×10^{8} m/s
Reduced Planck constant	$\hbar$	$1.054571817 \times 10^{-34}$ J·s
Gravitational constant	G	6.67430×10^{-11} m^3 kg^{-1} s^{-2}
Planck length	ℓ_{Pl}	1.616×10^{-35} m
Planck mass	M_{Pl}	2.176×10^{-8} kg
Characteristic length	L_0	$3.08567758 \times 10^{20}$ m (10 kpc)
Screening length	$\lambda_{\mathrm{screen}}$	5.082×10^{16} m (1.647 pc)
Effective coupling	β_{eff}	2.0×10^{7}
Kinetic coefficient	c_1	0.51
Kinetic coefficient sum	$c_1 + c_3$	0.83
Asymptotic field value	ν_0	0.911×10^{-3}
Present cosmic vacuum field value	ν_{now}	7.48×10^{-15}
Dimensionless present cosmic vacuum	$\tilde{\nu}_{\mathrm{now}}$	8.21×10^{-12}
Self-coupling magnitude	$\lvert\lambda\rvert$	7.37×10^{13}
Hubble constant	H_0	67.4 km/s/Mpc
Baryon density parameter	Ω_b	0.049
QFST-quanta density parameter	$\Omega_{\mathrm{QFST}}(0)$	0.25
Dark-energy density parameter	$\Omega_{\mathrm{DE}}(0)$	0.70
Redshift at equality	z_{eq}	3400
Density at equality	$\bar{\rho}_{\mathrm{eq}}$	1.057×10^{-16} kg/m^3
QFST quantum mass	m	3.88×10^{-24} eV

J. APPENDIX J: FLRW REDUCTION AND STRESS–ENERGY OF THE SPEED FIELD

This appendix provides a step-by-step derivation of the homogeneous (background) reduction of the FST vector sector and the corresponding definitions of energy density, pressure, and equation of state in a spatially flat FLRW spacetime.

J.1. Setup and conventions

We work with metric signature $(-,+,+,+)$ and a spatially flat FLRW line element

$$ds^2 = -c^2 dt^2 + a(t)^2\, \delta_{ij}\, dx^i dx^j, \tag{125}$$

with Hubble parameter $H \equiv \dot{a}/a$. The homogeneous timelike background of the speed field is taken as

$$\nu^\mu = (\nu(t), 0, 0, 0), \qquad \tilde{\nu}(t) = \frac{\nu(t)}{\nu_0}. \tag{126}$$

J.2. Covariant derivatives

The non-vanishing Christoffel symbols for Eq. (125) are

$$\Gamma^0_{ij} = \frac{a\dot{a}}{c^2}\, \delta_{ij}, \qquad \Gamma^i_{0j} = \Gamma^i_{j0} = H\, \delta^i_j. \tag{127}$$

Using Eq. (126), one finds

$$\nabla_0 \nu^0 = \dot{\nu}, \tag{128}$$
$$\nabla_i \nu^0 = 0, \tag{129}$$
$$\nabla_0 \nu^i = 0, \tag{130}$$
$$\nabla_i \nu^j = \Gamma^j_{i0}\, \nu^0 = H\nu\, \delta^j_i. \tag{131}$$

The divergence is therefore

$$\nabla_\mu \nu^\mu = \dot{\nu} + 3H\nu. \tag{132}$$

J.3. Reduction of the kinetic scalar

The kinetic scalar in Eq. (9) contains the three invariants

$$I_1 \equiv (\nabla_\mu \nu_\nu)(\nabla^\mu \nu^\nu), \tag{133}$$
$$I_2 \equiv (\nabla_\mu \nu^\mu)^2, \tag{134}$$
$$I_3 \equiv (\nabla_\mu \nu_\nu)(\nabla^\nu \nu^\mu). \tag{135}$$

For the homogeneous ansatz, Eqs. (128)–(131) imply

$$I_1 = \frac{\dot{\nu}^2}{c^2} + 3H^2\nu^2, \tag{136}$$
$$I_2 = (\dot{\nu} + 3H\nu)^2, \tag{137}$$
$$I_3 = \frac{\dot{\nu}^2}{c^2} + 3H^2\nu^2. \tag{138}$$

Hence the FLRW-reduced kinetic scalar is

$$\boxed{\mathcal{K}_{\mathsf{FLRW}} = L_0^2\left[(c_1+c_3)\left(\frac{\dot{\nu}^2}{c^2}+3H^2\nu^2\right) + c_2\left(\dot{\nu}+3H\nu\right)^2\right].} \tag{139}$$

This expression is dimensionless: $L_0^2\dot{\nu}^2/c^2$ and $L_0^2H^2\nu^2$ are both dimensionless.

J.4. Minisuperspace action and stress–energy

Inserting Eq. (139) into Eq. (8) yields the homogeneous action for the ν sector,

$$S_\nu = \int dt\, a(t)^3\left[\frac{c^4}{16\pi G L_0^2}\left(-\frac{1}{2}\mathcal{K}_{\mathsf{FLRW}} - V(\tilde{\nu})\right)\right]. \tag{140}$$

The stress–energy tensor is defined by

$$T^{(\nu)}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\left(\sqrt{-g}\,\mathcal{L}_V\right). \tag{141}$$

For a homogeneous and isotropic background, $T^{(\nu)}_{\mu\nu}$ takes the perfect-fluid form, and one defines

$$\rho_\nu(t) \equiv \frac{T^{(\nu)}_{00}}{c^2}, \qquad p_\nu(t) \equiv \frac{1}{3}T^{(\nu)}_{ij}g^{ij}. \tag{142}$$

To compute (ρ_ν, p_ν) explicitly, introduce a lapse $N(t)$ and write

$$ds^2 = -N(t)^2c^2dt^2 + a(t)^2\,\delta_{ij}\,dx^i dx^j, \tag{143}$$

so that $H = \dot{a}/(aN)$. The FLRW-reduced kinetic scalar generalizes to

$$\mathcal{K}_{\mathsf{FLRW}}(N) = L_0^2\left[(c_1+c_3)\left(\frac{1}{N^2}\frac{\dot{\nu}^2}{c^2}+3H^2\nu^2\right) + c_2\left(\frac{\dot{\nu}}{N}+3H\nu\right)^2\right]. \tag{144}$$

The minisuperspace Lagrangian is

$$L_\nu^{(\mathrm{mini})} = Na^3\frac{c^4}{16\pi G L_0^2}\left(-\frac{1}{2}\mathcal{K}_{\mathsf{FLRW}}(N) - V(\tilde{\nu})\right) \tag{145}$$

$$= Na^3\frac{c^4}{16\pi G L_0^2}\left(-\frac{1}{2}\mathcal{K}_{\mathsf{FLRW}}(N) + U(\tilde{\nu})\right), \tag{146}$$

where $U \equiv -V$ is defined in Eq. (11). Varying with respect to the lapse gives the Hamiltonian constraint. The corresponding energy density is

$$\rho_\nu = -\frac{1}{a^3}\frac{\partial L_\nu^{(\mathrm{mini})}}{\partial N}\bigg|_{N=1} = \frac{c^4}{16\pi G L_0^2}\left[\frac{1}{2}\mathcal{K}_{\mathsf{FLRW}}(N{=}1) + U(\tilde{\nu})\right]. \tag{147}$$

Using Eq. (139) this becomes

$$\boxed{\rho_\nu = \frac{c^4}{16\pi G L_0^2}\left\{\frac{L_0^2}{2}\left[(c_1+c_3)\left(\frac{\dot{\nu}^2}{c^2}+3H^2\nu^2\right) + c_2\left(\dot{\nu}+3H\nu\right)^2\right] + U(\tilde{\nu})\right\}}. \tag{148}$$

To obtain the pressure, vary with respect to $a(t)$ (equivalently g^{ij}). In minisuperspace form one finds

$$p_\nu = \frac{1}{3a^2}\frac{\partial L_\nu^{(\mathrm{mini})}}{\partial a}\bigg|_{N=1} = \frac{c^4}{16\pi G L_0^2}\left[-U(\tilde{\nu}) + \frac{1}{2}\mathcal{K}_{\mathsf{FLRW}}(N{=}1) - \frac{1}{3}H\frac{\partial \mathcal{K}_{\mathsf{FLRW}}}{\partial H}\bigg|_{N=1}\right]. \tag{149}$$

For Eq. (139), the derivative is

$$\frac{\partial \mathcal{K}_{\mathsf{FLRW}}}{\partial H} = 6L_0^2\left[(c_1+c_3)H\nu^2 + c_2\nu\left(\dot{\nu}+3H\nu\right)\right], \tag{150}$$

so that

$$p_\nu = \frac{c^4}{16\pi G L_0^2}\left\{-U(\tilde{\nu}) + \frac{L_0^2}{2}\left[(c_1+c_3)\left(\frac{\dot{\nu}^2}{c^2}+3H^2\nu^2\right) + c_2\left(\dot{\nu}+3H\nu\right)^2\right] - 2L_0^2H\left[(c_1+c_3)H\nu^2 + c_2\nu(\dot{\nu}+3H\nu)\right]\right\}. \tag{151}$$

Equations (148) and (151) define $w(t) = p_\nu/\rho_\nu$ exactly in terms of $(\nu, \dot{\nu}, H)$ and the logarithmic potential energy $U(\tilde{\nu})$.

J.5. Explicit background equation of motion for $\nu(t)$

The background equation of motion follows from the Euler–Lagrange equation applied to $L_\nu^{(\mathrm{mini})}$ in Eq. (146),

$$\frac{d}{dt}\left(\frac{\partial L_\nu^{(\mathrm{mini})}}{\partial \dot{\nu}}\right) - \frac{\partial L_\nu^{(\mathrm{mini})}}{\partial \nu} = 0. \tag{152}$$

Since $U = U(\tilde{\nu})$ with $\tilde{\nu} = \nu/\nu_0$, one has

$$\frac{dU}{d\nu} = \frac{1}{\nu_0}\frac{dU}{d\tilde{\nu}}, \qquad \frac{dU}{d\tilde{\nu}} = \frac{2V_0\tilde{\nu}/\nu_0^2}{1+\tilde{\nu}^2/\nu_0^2}. \tag{153}$$

For $N = 1$, the required derivatives of $\mathcal{K}_{\mathsf{FLRW}}$ (Eq. (139)) are

$$\frac{\partial \mathcal{K}_{\mathsf{FLRW}}}{\partial \dot{\nu}} = 2L_0^2\left[(c_1+c_3)\frac{\dot{\nu}}{c^2} + c_2\left(\dot{\nu}+3H\nu\right)\right], \tag{154}$$

$$\frac{\partial \mathcal{K}_{\mathsf{FLRW}}}{\partial \nu} = 6L_0^2\left[(c_1+c_3)H^2\nu + c_2H\left(\dot{\nu}+3H\nu\right)\right]. \tag{155}$$

Substituting into Eq. (152) gives the explicit homogeneous equation of motion

$$0 = \frac{d}{dt}\left[a^3\left((c_1+c_3)\frac{\dot{\nu}}{c^2}+c_2(\dot{\nu}+3H\nu)\right)\right] - 3a^3\left[(c_1+c_3)H^2\nu + c_2H(\dot{\nu}+3H\nu)\right] + \frac{a^3}{L_0^2}\frac{dU}{d\nu}. \tag{156}$$

J.6. Coupled background system (Friedmann + ν)

Let ρ_{tot} and p_{tot} be the total mass density and pressure of all components (baryons, radiation, and the ν sector). For a spatially flat universe the background expansion is governed by

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}}, \tag{157}$$

$$\dot{H} = -4\pi G\left(\rho_{\text{tot}} + \frac{p_{\text{tot}}}{c^2}\right), \tag{158}$$

with

$$\rho_{\text{tot}} = \rho_b + \rho_r + \rho_\nu, \qquad p_{\text{tot}} = p_r + p_\nu, \tag{159}$$

where (ρ_ν, p_ν) are given by Eqs. (148) and (151). Together with the ν equation of motion (156), these equations uniquely determine $a(t)$, $\nu(t)$, and hence $w(t)$ and $w(z)$.

J.7. Equation of state and the $w \simeq -1$ limit

The equation-of-state parameter is defined exactly as

$$\boxed{w(t) \equiv \frac{p_\nu(t)}{\rho_\nu(t)}}. \tag{160}$$

In the potential-dominated regime, where the effective kinetic contribution from Eq. (139) is small compared to $|V(\tilde{\nu})|$, one obtains $w \approx -1$ with deviations controlled by the ratio of the kinetic terms (including the $H\nu$ pieces) to the logarithmic potential.

DATA AVAILABILITY STATEMENT

The data used in this work are publicly available. The SPARC database [4] can be accessed at:

http://astroweb.cwru.edu/SPARC/

The classical FST paper (Paper 1) and the spacetime fabric paper (Paper 2) are available as preprints at:

- **FST: Galactic Dynamics** [1]:

  https://doi.org/10.20944/preprints202602.0601.v6

- **FST: Spacetime Fabric** [3]:

  https://www.preprints.org/manuscript/202604.0949/v3

The complete Python implementation of QFST, including verification codes and dimensional analysis, is permanently archived at:

https://doi.org/10.5281/zenodo.19387999

In addition, the numerical code used in this work is provided as supplementary files accompanying this manuscript.
All results in this paper can be reproduced by running the provided code with the SPARC database.

COMPETING INTERESTS STATEMENT

Not applicable (independent researcher).

ETHICS STATEMENT

Not applicable.

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