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Defining a new type of α-supra open soft sets using parametric supra topologies

Tareq M. Al-shami[1] *, S. Saleh[2] and Mohammed Jameel[1]

[1]**Department of Mathematics, Sana'a University, Sana'a, Yemen,**
[2]**Department of Mathematics, Hodeidah University, Hodeidah, Yemen.**

*Corresponding author: tareqalshami83@gmail.com

ABSTRACT

In this paper, we introduce novel concepts that represent unique contributions to the framework of supra soft topological spaces. First, we define and characterize the notion of feeble α-supra open soft subsets, establishing their fundamental properties. Through illustrative examples, we demonstrate the relationships between this class of soft subsets and existing generalizations of supra open soft sets. Then, we put forward the concepts of interior, closure, frontier, and accumulation operators induced by the class of feeble α-supra open soft and α-closed subsets, deriving their key properties and establishing fundamental relationships through some formulas.

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1. INTRODUCTION

To deal with day-to-day problems, containing uncertainty and vagueness, novel mathematical approaches have emerged to address them. Soft set theory, introduced by Molodtsov [1], is a significant advancement of these approaches. It represents a parameterized structure that effectively processes inherent uncertainties while overcoming the limitations of previous methods. The theoretical advantages of soft sets and their practical applications across various domains were established by Molodtsov's foundational work [1]. The theoretical framework was substantially advanced by Maji et al. [2] who suggested a soft set-based decision-making methodology. This work inspired many authors to explore various extensions and applications of soft sets. The operations and operators in soft set theory were first formalized in [3]; later work by [4] identified and corrected certain inconsistencies, aligning the soft set theory more closely with classical set theory. Subsequent work [5, 6] further expanded and improved the framework of soft set theory. Moreover, numerous authors have successfully integrated soft sets with other uncertainty models, particularly through hybridizations with fuzzy and rough set theories [7–10], which enhance the soft set theory's capacity for handling complex and uncertain systems.

In 2001, soft topology was independently introduced by Shabir and Naz [11] and Çağman et al. [12]. These approaches differed in their treatment of parameter sets inspite of they shared fundamental concepts starting from the method of defining soft topologies. Shabir and Naz stipulated a fixed set of parameters for each element of the soft topology, whereas Çağman et al. allowed parameter variation. Our work adopts Shabir and Naz's framework, which emphasizes the importance of constant parameter sets. Following these foundational contributions, soft topology rapidly attracted significant intellectual attention, reexamining classical topological concepts through this line of research. Min [13] profoundly discussed soft regular and soft normal spaces and established a classical systematic relationship that T_3-spaces imply T_3-spaces in soft topological spaces. El-Shafei et al. [14] proposed an additional classification of soft T_i-spaces that more faithfully preserved the properties of classical compactness and separation axioms. Further advancing this line of research, El-Shafei and Al-shami [15] introduced novel soft separation axioms and elucidated their connections with existing ones under

various circumstances. Subsequent types of soft separation axioms were presented by several researchers [16–18]. Al-shami [19] marked fresh soft separation axioms by using the idea of soft somewhat open sets, and then he illustrated how one applied to identify nutrition deficiencies of individuals.
Aygünoğlu and Aygün [20] presented the concepts of compact and Lindelöf spaces. Hida [21] described two sorts of soft compact spaces. Another kinds of compactness and Lindelöfness were defined using some generalizations of open soft and closed subsets such as soft regular closed [22] and soft somewhat open (somewhere dense) sets [23, 24]. The correspondence between some soft and crisp topological operators and properties was proved by Al-shami and Kočinac [25]. They inferred that many topological properties are transposable between enriched and extended soft topologies and their parametric topologies. Al-shami [26, 27] demonstrated practical applications of these theoretical advances, employing compactness and soft separation axioms to determine the missing values of information systems and optimize decision-making in tourism program selection. This application underscores the growing relevance of soft topological methods in solving real-world problems. Many classic topological concepts and ideas have been studied in soft topological spaces; for example: soft mappings between soft topological spaces [28], soft continuity [29], Menger spaces [30], maximal topologies [31] and expandable spaces [32], and generalizations of open soft sets [33–36].
Mashour et al. [37], in 1983, defined the concept of supra topologies and revealed its main properties. Then, many authors discussed topological properties via supra topological spaces; see, for example, [38–40]. In 2014, El-Sheikh et al. [41] introduced the notion of supra soft topologies as a generalization of supra topologies. This notion opens a door for researchers and scholars to investigate topological properties via supra soft topologies. For example, some forms of supracontinuity and the decomposition of soft supraclosed sets were studied by Abd El-latif [42] and Shaaban [43]. Azzam et al. [44] suggested an approach to generate soft topologies. Further contributions on supra soft topologies concerning covering and connectedness properties were made by [45, 46].
The motivations for presenting this study are, first, to create a novel approach to generalizing supra soft topology by using its classical supra topologies. The suggested approach generalized its counterparts introduced in soft topologies [47–51]. Second, to present a new framework for generating soft topological concepts, like soft operators, which we achieve herein. The scholars and researchers can explore further ideas utilizing the proposed class of feeble α-supra open soft sets like soft continuity, separation axioms, and covering properties. A final objective is to highlight the value of soft topology by constructing analogs for various crisp topological concepts.
This manuscript is structured as follows. Section 2 compiles the requisite definitions and preliminary results for a self-contained exposition. Section 3 presents a novel generalization of supra open soft sets, which we term feeble supra soft α-sets. We analyze their principal features and illustrate them with examples. Building upon this foundation, Section 4 employs the new notion to define and interrelate several key concepts: the feeble α-supra soft interior, closure, frontier, and accumulation points.

2. PRELIMINARIES

This section reviews the essential background and notation for this work.

2.1. DEFINITION

[1] Let Z and $\triangle$ be sets of objects and parameters, respectively. A set-valued mapping $C : \triangle \to 2^{Z}$ is named a soft set (S-set, in short); it is denoted by the ordered pair $(C, \triangle)$.
We use a pair $(C, \triangle)$ to denote a S-set $(C, \triangle)$ and represent as follows

$$(C, \triangle) = \{(p, C(p)) : p \in \triangle \text{ and } C(p) \in 2^{Z}\};$$

The class of all S-sets defined on Z with respect to the set of parameters $\triangle$ is denoted by $2^{Z_{\triangle}}$.

2.2. DEFINITION

[3, 52] We call $(C, \triangle)$ absolute (resp., null, pseudo constant) S-set, symbolized by $\widetilde{Z}$ (resp., ϕ, $\widetilde{Z} * \phi$), if $C(p) = Z$ (resp., $C(p) = \emptyset$, $C(p) = Z$ or $\emptyset$) for all $p \in \triangle$. Also, $(C, \triangle)$, with the property "$C(p) = \{z\}$ for a fixed parameter p and $C(a) = \emptyset$ for all $a \in \triangle - \{p\}$", is named a soft point; it denoted by z_p. We write $z_p \in (C, \triangle)$ if $z \in C(p)$.

2.3. DEFINITION

[53] $(C, \triangle)$ is subset of $(O, \triangle)$, symbolized by $(C, \triangle)\widetilde{\subseteq}(O, \triangle)$ if for each $p \in \triangle$ the relation $C(p) \subseteq O(p)$ holds.

2.4. DEFINITION

[4] If for all $p \in \triangle$ we have $O(p) = Z - C(p)$, then $(O, \triangle)$ is referred as a complement of $(C, \triangle)$; it is singed by $(C, \triangle)^c = (C^c, \triangle)$.

2.5. DEFINITION

[5] Let $(C, \triangle)$ and $(O, \triangle)$ be S-sets. Then:

(i) $(C, \triangle)\widetilde{\bigcup}(O, \triangle) = (H, \triangle)$ s.t. $H(p) = C(p) \bigcup O(p)$ for all $p \in \triangle$.

(ii) $(C,\triangle)\widetilde{\bigcap}(O,\triangle) = (H,\triangle)$ s.t. $H(p) = C(p)\bigcap O(p)$ for all $p \in \triangle$.
(iii) $(C,\triangle) \setminus (O,\triangle) = (H,\triangle)$ s.t. $H(p) = C(p) \setminus O(p)$ for all $p \in \triangle$.
(iv) $(C,\triangle) \times (O,\triangle) = (H,\triangle)$ s.t. $H(p_1,p_2) = C(p_1) \times O(p_2)$ for all $(p_1,p_2) \in \triangle \times \triangle$.

The adjusted version of the definition of soft mappings is given in the following.

2.6. DEFINITION

[54] Let $\mathbb{H} : \mathsf{Z} \to \mathsf{Y}$ and $\pi : \triangle \to \mathbb{Q}$ be crisp mappings. A soft mapping $\mathbb{H}_\pi$ of $2^{\mathsf{Z}_\triangle}$ into $2^{\mathsf{Y}_\mathbb{Q}}$ is a relation such that each $\mathsf{z}_p \in 2^{\mathsf{Z}_\triangle}$ is related to one and only one $\mathsf{y}_q \in 2^{\mathsf{Y}_\mathbb{Q}}$ such that

$$\mathbb{H}_\pi(\mathsf{z}_p) = \mathbb{H}(\mathsf{z})_{\pi(p)} \text{ for all } \mathsf{z}_p \in 2^{\mathsf{Z}_\triangle}.$$

In addition, $\mathbb{H}_\pi^{-1}(\mathsf{y}_q) = \widetilde{\bigcup}_{\substack{\mathsf{z}\in\mathbb{H}^{-1}(\mathsf{y}) \\ p\in\pi^{-1}(q)}} \mathsf{z}_p$ for each $\mathsf{y}_q \in 2^{\mathsf{Y}_\mathbb{Q}}$.
That is, the image of $(C,\triangle)$ and pre-image of $(O,\mathbb{Q})$ under a soft mapping $\mathbb{H}_\pi$: $2^{\mathsf{Z}_\triangle} \to 2^{\mathsf{Y}_\mathbb{Q}}$ are respectively given by:

$$\mathbb{H}_\pi(C,\triangle) = \widetilde{\bigcup}_{\mathsf{z}_p\in(C,\triangle)}\mathbb{H}_\pi(\mathsf{z}_p,),$$

and

$$\mathbb{H}_\pi^{-1}(O,\mathbb{Q}) = \widetilde{\bigcup}_{\mathsf{y}_q\in(O,\mathbb{Q})}\mathbb{H}_\pi^{-1}(\mathsf{y}_q).$$

A soft mapping is described as surjective (resp., injective, bijective) if its two crisp mappings satisfy this description.

2.7. THEOREM

[28] Let $\mathbb{H}_\pi$: $2^{\mathsf{Z}_\triangle} \to 2^{\mathsf{Y}_\mathbb{Q}}$ be a soft mapping and let $(C,\triangle)$ and $(O,\triangle)$ respectively be subsets of $\widetilde{\mathsf{Z}}$ and $\widetilde{\mathsf{Y}}$. Then

(i) $(C,\triangle)\widetilde{\subseteq}\mathbb{H}_\pi^{-1}(\mathbb{H}_\pi(C,\triangle))$.
(ii) $(C,\triangle) = \mathbb{H}_\pi^{-1}(\mathbb{H}_\pi(C,\triangle))$ if $\mathbb{H}_\pi$ is injective.
(iii) $\mathbb{H}_\pi(\mathbb{H}_\pi^{-1}(O,\mathbb{Q}))\widetilde{\subseteq}(O,\mathbb{Q})$.
(iv) $\mathbb{H}_\pi(\mathbb{H}_\pi^{-1}(O,\mathbb{Q})) = (O,\mathbb{Q})$ if $\mathbb{H}_\pi$ is surjective.

2.8. DEFINITION

[1] A subfamily θ of $2^{\mathsf{Z}_\triangle}$ is said to be a supra soft topology if the following terms are satisfied:

(i) $\widetilde{\mathsf{Z}}$ and ϕ are elements of θ.
(ii) θ is closed under the arbitrary unions.

We use the symbol $(\mathsf{Z},\theta,\triangle)$ to refer to a supra soft topological space (briefly, supra soft T-space). The terms of the supra open S-sets and the supra closed S-sets are given for the elements in θ and their complements, respectively.

2.9. DEFINITION

[1] For a S-subset $(C,\triangle)$ of a supra soft T-space $(\mathsf{Z},\theta,\triangle)$, the supra soft interior and supra soft closure of $(C,\triangle)$, denoted respectively by $int(C,\triangle)$ and $cl(C,\triangle)$, are defined as follows:

(i) $int(C,\triangle) = \widetilde{\bigcup}\{(O,\triangle) \in \theta : (O,\triangle)\widetilde{\subseteq}(C,\triangle)\}$.
(ii) $cl(C,\triangle) = \widetilde{\bigcap}\{(H,\triangle) : (C,\triangle)\widetilde{\subseteq}(H,\triangle)$ and $(H^c,\triangle) \in \theta\}$.

2.10. THEOREM

[1] Let $(\mathsf{Z},\theta,\triangle)$ be a supra soft T-space. Then

$$\theta_p = \{C(p) : (C,\triangle) \in \theta\}$$

is a supra topology on Z for every $p \in \triangle$. We will call this supra topology a parametric supra topology.

2.11. DEFINITION

A S-subset $(C,\triangle)$ of supra soft T-space $(\mathsf{Z},\theta,\triangle)$ is said to be:

(i) α-supra open soft [33] if $(C,\triangle)\widetilde{\subseteq}int(cl(int(C,\triangle)))$.
(ii) semi-supra open soft [**20**] if $(C,\triangle)\widetilde{\subseteq}cl(int(C,\triangle))$.
(iii) β-supra open soft if $(C,\triangle)\widetilde{\subseteq}cl(int(cl(C,\triangle)))$.
(iv) sw-supra open soft[55] if $(C,\triangle) = \phi$ or $int(C,\triangle) \neq \phi$.

2.12. DEFINITION

[29] A soft mapping $\mathbb{H}_\pi : (\mathsf{Z},\theta_\mathsf{Z},\triangle) \to (\mathsf{Y},\theta_\mathsf{Y},\triangle)$ is said to be supra soft continuous if $\mathbb{H}_\pi^{-1}(C,\triangle)$ is a supra open S-set where $(C,\triangle)$ is supra open soft.

3. FEEBLE α-SUPRA OPEN S-SETS AND THEIR MAIN FEATURES

The central concept of this article, termed feeble α-supra open S-sets, is introduced in this section. It is shown that this family of S-subsets constitutes a novel extension of supra open S-sets, positioned strictly between α-supra open soft and supra soft sw-open subsets within the extended soft topological framework. Furthermore, certain divergences between this class and other extensions are highlighted through counterexamples, demonstrating, for instance, that it is not closed under soft union. Among the other results presented, the behavior of this family w.r.t. fundamental topological properties and under products of soft spaces is investigated.

3.1. DEFINITION

We call $(C,\triangle)$ in a supra soft T-space $(\mathsf{Z},\theta,\triangle)$ a feeble α-supra open S-set if

$$C(p) = \emptyset \text{ for all } p \in \triangle$$

or

$$\emptyset \neq C(p) \subseteq int(cl(int(C(p))))$$

for some $p \in \triangle$

In other words, if $(C, \triangle)$ is null or some of its nonempty components are α-supra open set.

We call $(C, \triangle)$ a feeble α-supra closed S-set if $(C^c, \triangle)$ is feeble α-supra open soft.

3.2. THEOREM

A set $(C, \triangle)$ in a supra soft T-space $(Z, \theta, \triangle)$ is feeble α-supra closed soft iff $(C, \triangle) = \widetilde{Z}$ or

$$cl(int(cl(C(p)))) \subseteq C(p) \neq Z$$

for some $p \in \triangle$.

Necessity: Let $(C, \triangle)$ be a feeble α-supra closed S-set. Then,

$$(C^c, \triangle) = \phi,$$

or

$$\emptyset \neq C^c(p) \subseteq int(cl(int(C^c(p)))),$$

for some $p \in \triangle$. Accordingly, we obtain either

$$(C, \triangle) = \widetilde{Z},$$

or

$$cl(int(cl(C(p)))) \subseteq C(p) \neq Z,$$

for some $p \in \triangle$, which proves this implication.

Sufficiency: Let $(C, \triangle)$ be a S-set such that

$$(C, \triangle) = \widetilde{Z},$$

or

$$cl(int(cl(C(p)))) \subseteq C(p) \neq Z,$$

for some $p \in \triangle$. Then,

$$(C^c, \triangle) = \phi,$$

or

$$\emptyset \neq C^c(p) \subseteq int(cl(int(C^c(p)))),$$

for some $p \in \triangle$. This implies that $(C^c, \triangle)$ is feeble α-supra open soft. Hence, $(C, \triangle)$ is feeble α-supra closed soft, as required.

The following counterexample proves that the class of feeble α-supra open soft (equivalently, α-closed) sets is neither closed under soft union nor under soft intersection.

3.3. EXAMPLE

Let $Z = \{z_1, z_2, z_3\}$ be the universe and $\triangle = \{p_1, p_2\}$ be a set of parameters. Consider the class θ consisting of ϕ, $\widetilde{Z}$ and S-sets over Z with $\triangle$ as follows:

$$(C_1, \triangle) = \{(p_1, Z), (p_2, \{z_1\})\};$$

$$(C_2, \triangle) = \{(p_1, \{z_1\}), (p_2, Z)\};$$

$$(C_3, \triangle) = \{(p_1, \{z_2\}), (p_2, \{z_3\})\};$$

$$(C_4, \triangle) = \{(p_1, \{z_1, z_2\}), (p_2, Z)\};$$

and

$$(C_5, \triangle) = \{(p_1, Z), (p_2, \{z_1, z_3\})\}.$$

Then, $(Z, \theta, \triangle)$ is a supra soft T-space. Now we have the following:

$$(E, \triangle) = \{(p_1, \{z_1\}), (p_2, \{z_2\})\};$$

$$(C, \triangle) = \{(p_1, \{z_3\}), (p_2, \{z_1\})\};$$

$$(O, \triangle) = \{(p_1, Z), (p_2, \{z_1, z_2\})\} \text{ and}$$

$$(H, \triangle) = \{(p_1, \{z_1, z_3\}), (p_2, Z)\}.$$

are feeble α-supra open S-sets. Note that neither the union of $(E, \triangle)$ and $(C, \triangle)$ is a feeble α-supra open S-set nor the intersection of $(O, \triangle)$ and $(H, \triangle)$ is a feeble α-supra open S-set because neither $\{z_1, z_3\}$ is an α-supra open subset of (Z, θ_{p_1}) nor $\{z_1, z_2\}$ is an α-supra open subset of (Z, θ_{p_2}).

3.4. REMARK

If $(C, \triangle)$ is pseudo constant S-subset, then it is a feeble α-supra S-subset because either $int(cl(int(C(p)))) = Z$ for some $p \in \triangle$ or $C(p) = \emptyset$ for all $p \in \triangle$.

The proofs of the next propositions are easy, so we omit.

3.5. THEOREM

Every supra open S-set is feeble α-supra open soft.

3.6. THEOREM

A S-set $(C, \triangle)$ of a supra soft T-space $(Z, \theta, \triangle)$ with $C(p) = Z$ (resp., $C(p) = \emptyset$) is feeble α-supra open soft (resp., feeble α-supra closed soft).

Note that the converse of the above two propositions are false, in general, as illustrated by Example 3.3.

3.7. THEOREM

Let $\mathbb{H}_\pi$: $(Z, \theta, \triangle) \to (Y, \mu, Q)$ be a soft mapping such that $\mathbb{H}$: $(Z, \theta_p) \to (Y, \mu_{\pi(p)=q})$ is a supra bicontinuous mapping for each p and π is injective. Then, the image of feeble α-supra open S-set is feeble α-supra open S-set.

Let $(C, \triangle)$ be a feeble α-supra S-subset of a supra soft T-space $(Z, \theta, \triangle)$. Then, there is $p \in \triangle$ such that $C(p)$ is a nonempty α-open subset. Assume $\pi(p) = q$. By hypothesis of supra bicontinuity of $\mathbb{H}$: $(Z, \theta_p) \to (Y, \mu_{\pi(p)=q})$ we have for each subset V of Z: $\mathbb{H}(cl(V)) \subseteq cl(\mathbb{H}(V))$ (by supra continuity) and $\mathbb{H}(int(V)) \subseteq int(\mathbb{H}(V))$ (by supra open). This implies that

$$\mathbb{H}(C(p)) \widetilde{\subseteq} \mathbb{H}(int(cl(int(C(p))))) \widetilde{\subseteq} int(cl(int(\mathbb{H}(C(p))))).$$

Accordingly, $\mathbb{H}(C(p))$ is a nonempty α-supra open component of $\mathbb{H}_{\pi}(C,\triangle)$. Thus, $\mathbb{H}_{\pi}(C,\triangle)$ is a feeble α-supra open S-subset of (Y,μ,Q).

3.8. COROLLARY

A feeble α-supra open S-set is a supra topological property.

4. FEEBLE α-SUPRA INTERIOR AND FEEBLE α-SUPRA CLOSURE OPERATORS

In this part, the concepts of interior, closure, frontier, and accumulation operators are constructed from the classes of feeble α-supra open soft and feeble α-supra closed S-sets. Their fundamental properties are established, and the relationships among them are examined. Furthermore, through counterexamples, it is demonstrated that the feeble α-supra interior (resp., closure) of a S-set fails to be a feeble α-supra open (resp., closed) set.

4.1. DEFINITION

The feeble α-supra interior points of a subset $(C,\triangle)$ of a supra soft T-space $(Z,\theta,\triangle)$, denoted by $int^{\alpha}(C,\triangle)$, is defined as the union of all feeble α-supra open S-sets contained in $(C,\triangle)$.
Note that the feeble α-supra interior points of a subset need not be a feeble α-supra open set. In other words, $(C,\triangle)$ may fail to be feeble α-supra open set even if it equals $int^{\alpha}(C,\triangle)$.
We leave the proofs of the next propositions to the reader, as they are direct consequences of the definitions.

4.2. THEOREM

Let $(C,\triangle)$ be a S-subset of a supra soft T-space $(Z,\theta,\triangle)$ and $z_p \in \widetilde{Z}$. Then $z_p \in int^{\alpha}(C,\triangle)$ iff there is a feeble α-supra open S-set $(O,\triangle)$ contains z_p such that $(O,\triangle)\widetilde{\subseteq}(C,\triangle)$.

4.3. THEOREM

Let $(C,\triangle)$ and $(O,\triangle)$ be S-subsets of a supra soft T-space $(Z,\theta,\triangle)$. Then

(i) $int^{\alpha}(C,\triangle)\widetilde{\subseteq}(C,\triangle)$.
(ii) if $(C,\triangle)\widetilde{\subseteq}(O,\triangle)$, then $int^{\alpha}(C,\triangle)\widetilde{\subseteq}int^{\alpha}(O,\triangle)$.

4.4. COROLLARY

For any S-subsets $(C,\triangle)$, $(O,\triangle)$ of a supra soft T-space $(Z,\theta,\triangle)$, we have the following results:

(i) $int^{\alpha}[(C,\triangle)\widetilde{\bigcap}(O,\triangle)]\widetilde{\subseteq}int^{\alpha}(C,\triangle)\widetilde{\bigcap}int^{\alpha}(O,\triangle)$.
(ii) $int^{\alpha}(C,\triangle)\widetilde{\bigcup}int^{\alpha}(O,\triangle)\widetilde{\subseteq}int^{\alpha}[(C,\triangle)\widetilde{\bigcup}(O,\triangle)]$.

It is a direct consequence of the following relations

(i) $(C,\triangle)\widetilde{\bigcap}(O,\triangle)\widetilde{\subseteq}(C,\triangle)$ and $(C,\triangle)\widetilde{\bigcap}(O,\triangle)\widetilde{\subseteq}(O,\triangle)$.
(ii) $(C,\triangle)\widetilde{\subseteq}[(C,\triangle)\widetilde{\bigcup}(O,\triangle)]$ and $(O,\triangle)\widetilde{\subseteq}[(C,\triangle)\widetilde{\bigcup}(O,\triangle)]$

Note that in Theorem 4.3 and Corollary 4.4 the inclusion relations are proper.

4.5. DEFINITION

The feeble α-supra closure points of a subset $(C,\triangle)$ of a supra soft T-space $(Z,\theta,\triangle)$, denoted by $cl^{\alpha}(C,\triangle)$, is defined as the intersection of all feeble α-supra closed S-sets containing $(C,\triangle)$.
It can be seen that the feeble α-supra closure points of a set are not always a feeble supra α-closed set. Therefore, a soft set satisfying $cl^{\alpha}(C,\triangle)=(C,\triangle)$ is not necessarily $(C,\triangle)$ is a feeble supra α-closed set.

4.6. THEOREM

Let $(C,\triangle)$ be a subset of a supra soft T-space $(Z,\theta,\triangle)$ and $z_p \in \widetilde{Z}$. Then $z_p \in cl^{\alpha}(C,\triangle)$ iff $(O,\triangle)\widetilde{\bigcap}(C,\triangle)\neq\phi$ for each feeble α-supra open S-set $(O,\triangle)$ contains z_p.

[$\Rightarrow$] Let $z_p \in cl^{\alpha}(C,\triangle)$. Suppose that there is feeble α-supra open S-set $(O,\triangle)$ containing z_p with

$$(O,\triangle)\widetilde{\bigcap}(C,\triangle)=\phi.$$

Then

$$(C,\triangle)\widetilde{\subseteq}(O^c,\triangle).$$

Therefore,

$$cl^{\alpha}(C,\triangle)\widetilde{\subseteq}(O^c,\triangle).$$

Thus

$$z_p \notin cl^{\alpha}(C,\triangle).$$

But this is a contradiction, so

$$(O,\triangle)\widetilde{\bigcap}(C,\triangle)\neq\phi \text{ holds.}$$

[$\Leftarrow$] Let

$$(O,\triangle)\widetilde{\bigcap}(C,\triangle)\neq\phi$$

for each feeble α-supra open S-set $(O,\triangle)$ contains z_p. Let us assume that

$$z_p \notin cl^{\alpha}(C,\triangle).$$

Then there is a feeble α-supra closed S-set $(H,\triangle)$ containing $(C,\triangle)$ with $z_p \notin (H,\triangle)$. So

$$z_p \in (H^c,\triangle)$$

and

$$(H^c,\triangle)\widetilde{\bigcap}(C,\triangle)=\phi.$$

But this contradicts our assumption. We have thus proved the claim

4.7. COROLLARY

If

$$(C,\triangle)\widetilde{\bigcap}(O,\triangle)=\phi$$

such that $(C,\triangle)$ is a feeble α-supra open S-set and $(O,\triangle)$ is a S-set in $(\mathrm{Z},\theta,\triangle)$, then

$$(C,\triangle)\widetilde{\bigcap}cl^{\alpha}(O,\triangle)=\phi.$$

Straightforward.

4.8. THEOREM

For a subset $(C,\triangle)$ of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$, the next results hold true.

(i) $[int^{\alpha}(C,\triangle)]^{c}=cl^{\alpha}(C^{c},\triangle)$.
(ii) $[cl^{\alpha}(C,\triangle)]^{c}=int^{\alpha}(C^{c},\triangle)$.

(i) If

$$z_{p}\notin[int^{\alpha}(C,\triangle)]^{c},$$

then there is a feeble α-supra open S-set $(O,\triangle)$ with

$$z_{p}\in(O,\triangle)\tilde{\subseteq}(C,\triangle).$$

Therefore,

$$(C^{c},\triangle)\widetilde{\bigcap}(O,\triangle)=\phi,$$

and hence,

$$z_{p}\notin cl^{\alpha}(C^{c},\triangle).$$

Conversely, if $z_{p}\notin cl^{\alpha}(C^{c},\triangle)$ one may verify $z_{p}\notin[int^{\alpha}(C,\triangle)]^{c}$ by adapting the previous steps.
(ii) The proof follows an argument similar to (i).
The following proposition is presented without proof, as it follows directly from the definitions.

4.9. THEOREM

Let $(C,\triangle)$, $(O,\triangle)$ be S-subsets of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$. Then

(i) $(C,\triangle)\tilde{\subseteq}cl^{\alpha}(C,\triangle)$.
(ii) if $(C,\triangle)\tilde{\subseteq}(O,\triangle)$, then $cl^{\alpha}(C,\triangle)\tilde{\subseteq}cl^{\alpha}(O,\triangle)$.

4.10. COROLLARY

The next properties hold for all subsets of $(C,\triangle)$, $(O,\triangle)$ of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$.

(i) $cl^{\alpha}[(C,\triangle)\tilde{\cap}(O,\triangle)]\tilde{\subseteq}cl^{\alpha}(C,\triangle)\tilde{\cap}cl^{\alpha}(O,\triangle)$.
(ii) $cl^{\alpha}(C,\triangle)\tilde{\cup}cl^{\alpha}(O,\triangle)\tilde{\subseteq}cl^{\alpha}[(C,\triangle)\tilde{\cup}(O,\triangle)]$.

It automatically comes from the following:

(i) $(C,\triangle)\tilde{\cap}(O,\triangle)\tilde{\subseteq}(C,\triangle)$ and $(C,\triangle)\tilde{\cap}(O,\triangle)\tilde{\subseteq}(O,\triangle)$.

(ii) $(C,\triangle)\tilde{\subseteq}[(C,\triangle)\tilde{\cup}(O,\triangle)]$ and $(O,\triangle)\tilde{\subseteq}[(C,\triangle)\tilde{\cup}(O,\triangle)]$.

Note that, in Theorem 4.9 and Corollary 4.10, the inclusion relations are proper.

4.11. DEFINITION

A soft point z_{p} is said to be a feeble α-supra frontier point of a subset $(C,\triangle)$ of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$ if z_{p} belongs to the complement of $int^{\alpha}(C,\triangle)\widetilde{\bigcup}int^{\alpha}(C^{c},\triangle)$.

All feeble α-supra frontier points of $(C,\triangle)$ is called a feeble α-supra frontier set, denoted by $f^{\alpha}(C,\triangle)$.

4.12. THEOREM

$$f^{\alpha}(C,\triangle)=cl^{\alpha}(C,\triangle)\widetilde{\bigcap}cl^{\alpha}(C^{c},\triangle)$$

for every subset $(C,\triangle)$ of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$.

$$\begin{aligned}
f^{\alpha}(C,\triangle) &= [int^{\alpha}(C,\triangle)\widetilde{\bigcup}int^{\alpha}(C^{c},\triangle)]^{c}\\
&= [int^{\alpha}(C,\triangle)]^{c}\widetilde{\bigcap}[int^{\alpha}(C^{c},\triangle)]^{c} \quad (De\ \ Morgan's\ \ law)\\
&= cl^{\alpha}(C^{c},\triangle)\widetilde{\bigcap}cl^{\alpha}(C,\triangle) \quad (Theorem\ \ 4.8(ii))
\end{aligned}$$

4.13. COROLLARY

For every subset $(C,\triangle)$ of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$, the following properties hold.

(i) $f^{\alpha}(C,\triangle)=f^{\alpha}(C^{c},\triangle)$.
(ii) $f^{\alpha}(C,\triangle)=cl^{\alpha}(C,\triangle)\setminus int^{\alpha}(C,\triangle)$.
(iii) $cl^{\alpha}(C,\triangle)=int^{\alpha}(C,\triangle)\tilde{\bigcup}f^{\alpha}(C,\triangle)$.
(iv) $int^{\alpha}(C,\triangle)=(C,\triangle)\setminus f^{\alpha}(C,\triangle)$.

(i) Obvious.

(ii) $f^{\alpha}(C,\triangle)=cl^{\alpha}(C,\triangle)\tilde{\cap}cl^{\alpha}(C^{c},\triangle)=cl^{\alpha}(C,\triangle)\setminus[cl^{\alpha}(C^{c},\triangle)]^{c}$. By (ii) of Theorem 4.8, the desired relation follows.

(iii) $int^{\alpha}(C,\triangle)\tilde{\bigcup}f^{\alpha}(C,\triangle)=int^{\alpha}(C,\triangle)\tilde{\bigcup}[cl^{\alpha}(C,\triangle)\setminus int^{\alpha}(C,\triangle)]=cl^{\alpha}(C,\triangle)$.

(iv)

$$\begin{aligned}
(C,\triangle)\setminus f^{\alpha}(C,\triangle) &= (C,\triangle)\setminus[cl^{\alpha}(C,\triangle)\setminus int^{\alpha}(C,\triangle)]\\
&= (C,\triangle)\widetilde{\bigcap}[cl^{\alpha}(C,\triangle)\widetilde{\bigcap}(int^{\alpha}(C,\triangle))^{c}]^{c}\\
&= (C,\triangle)\widetilde{\bigcap}[(cl^{\alpha}(C,\triangle))^{c}\widetilde{\bigcup}int^{\alpha}(C,\triangle)]\\
&= [(C,\triangle)\widetilde{\bigcap}(cl^{\alpha}(C,\triangle))^{c}]\widetilde{\bigcup}[(C,\triangle)\widetilde{\bigcap}int^{\alpha}(C,\\
&= int^{\alpha}(C,\triangle).
\end{aligned}$$

4.14. THEOREM

Let $(C,\triangle),(O,\triangle)$ be subsets of a supra soft T-space $(\mathrm{Z},\theta,\triangle)$, the following properties hold.

(i) $f^{\alpha}(int^{\alpha}(C,\triangle))\tilde{\subseteq}f^{\alpha}(C,\triangle)$.
(ii) $f^{\alpha}(cl^{\alpha}(C,\triangle))\tilde{\subseteq}f^{\alpha}(C,\triangle)$.

By substituting in the formula No. (iii) of Corollary 4.13, the proof follows.

4.15. THEOREM

Let $(C,\triangle)$ be a subset of a supra soft T-space $(Z,\theta,\triangle)$. Then

(i) $(C,\triangle)=int^{\alpha}(C,\triangle)$ iff $f^{\alpha}(C,\triangle)\widetilde{\bigcap}(C,\triangle)=\phi$.
(ii) $(C,\triangle)=cl^{\alpha}(C,\triangle)$ iff $f^{\alpha}(C,\triangle)\widetilde{\subseteq}(C,\triangle)$.

(i) Suppose that

$$(C,\triangle)=int^{\alpha}(C,\triangle).$$

Then by (iv) of Corollary 4.13,

$$(C,\triangle)=int^{\alpha}(C,\triangle)=(C,\triangle)\setminus f^{\alpha}(C,\triangle)$$

and hence,

$$f^{\alpha}(C,\triangle)\widetilde{\bigcap}(C,\triangle)=\phi.$$

Conversely, let $z_p\in(C,\triangle)$. Since $z_p\notin f^{\alpha}(C,\triangle)$ and $z_p\in cl^{\alpha}(C,\triangle)$, by (iii) of Corollary 4.13, $z_p\in int^{\alpha}(C,\triangle)$. Therefore,

$$int^{\alpha}(C,\triangle)=(C,\triangle),$$

which establishes the claim.
(ii) Assume that

$$(C,\triangle)=cl^{\alpha}(C,\triangle).$$

Then

$$f^{\alpha}(C,\triangle)=cl^{\alpha}(C,\triangle)\widetilde{\bigcap}cl^{\alpha}(C^{c},\triangle)\widetilde{\subseteq}cl^{\alpha}(C,\triangle)=(C,\triangle),$$

which establishes the claim.
Conversely, if $f^{\alpha}(C,\triangle)\widetilde{\subseteq}(C,\triangle)$, then by (iii) of Corollary 4.13,

$$cl^{\alpha}(C,\triangle)\widetilde{\subseteq}int^{\alpha}(C,\triangle)\widetilde{\bigcup}(C,\triangle)=(C,\triangle)$$

and hence

$$cl^{\alpha}(C,\triangle)=(C,\triangle),$$

as required.

4.16. COROLLARY

Let $(C,\triangle)$ be a subset of a supra soft T-space $(Z,\theta,\triangle)$. Then

$$int^{\alpha}(C,\triangle)=(C,\triangle)=cl^{\alpha}(C,\triangle)$$

iff

$$f^{\alpha}(C,\triangle)=\phi.$$

4.17. DEFINITION

A soft point z_p is said to be a feeble α-supra accumulation point of a subset $(C,\triangle)$ of a supra soft T-space $(Z,\theta,\triangle)$ if

$$[(O,\triangle)\setminus z_p]\bigcap(C,\triangle)\neq\phi$$

for each feeble α-supra open S-set $(O,\triangle)$ containing z_p. All feeble α-supra accumulation points of $(C,\triangle)$ is called a feeble supra α-derived set and denoted by $l^{\alpha}(C,\triangle)$.

4.18. THEOREM

Let $(C,\triangle)$ and $(O,\triangle)$ be subsets of a supra soft T-space $(Z,\theta,\triangle)$. If $(C,\triangle)\widetilde{\subseteq}(O,\triangle)$, then $l^{\alpha}(C,\triangle)\widetilde{\subseteq}l^{\alpha}(O,\triangle)$.
Straightforward by Definition 4.17.

4.19. COROLLARY

Consider $(C,\triangle)$ and $(O,\triangle)$ are subsets of a supra soft T-space $(Z,\theta,\triangle)$. Then:

(i) $l^{\alpha}[(C,\triangle)\widetilde{\bigcap}(O,\triangle)]\widetilde{\subseteq}l^{\alpha}(C,\triangle)\widetilde{\bigcap}\,l^{\alpha}(O,\triangle)$.
(ii) $l^{\alpha}(C,\triangle)\widetilde{\bigcup}\,l^{\alpha}(O,\triangle)\widetilde{\subseteq}l^{\alpha}[(C,\triangle)\widetilde{\bigcup}(O,\triangle)]$.

4.20. THEOREM

Let $(C,\triangle)$ be a subset of a supra soft T-space $(Z,\theta,\triangle)$, then

$$cl^{\alpha}(C,\triangle)=(C,\triangle)\widetilde{\bigcup}\,l^{\alpha}(C,\triangle).$$

The side

$$(C,\triangle)\widetilde{\bigcup}\,l^{\alpha}(C,\triangle)\widetilde{\subseteq}cl^{\alpha}(C,\triangle)$$

is clear. To verify the opposite implication, let

$$z_p\notin[(C,\triangle)\widetilde{\bigcup}l^{\alpha}(C,\triangle)].$$

Then $z_p\notin(C,\triangle)$ and $z_p\notin l^{\alpha}(C,\triangle)$. Therefore, there is feeble α-supra open soft $(O,\triangle)$ containing z_p with

$$(O,\triangle)\widetilde{\cap}(C,\triangle)=\phi.$$

Thus, $z_p\notin cl^{\alpha}(C,\triangle)$. Hence, we find that

$$cl^{\alpha}(C,\triangle)=(C,\triangle)\widetilde{\bigcup}l^{\alpha}(C,\triangle).$$

4.21. COROLLARY

Let $(C,\triangle)$ be a feeble α-supra closed S-subset of a supra soft T-space $(Z,\theta,\triangle)$, then $l^{\alpha}(C,\triangle)\widetilde{\subseteq}(C,\triangle)$.

5. CONCLUSION

Supra soft topological spaces form one of the generalizations of soft topological spaces; they are defined by ignoring the topological condition of finite intersection. This paper has investigated one of the extensions of supra open sets in supra soft topological spaces, namely, feeble α-supra open soft sets. We have derived their basic properties and discovered the relationships between them and some generalizations of supra open soft sets introduced in published works. We also employed feeble α-supra open and feeble α-closed soft sets to define the concepts of interior, closure, frontier, and accumulation operators. We have explored their key properties and established fundamental relationships through some formulas.

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