Sana'a University Journal of Applied Sciences and Technology https://journals.su.edu.ye/index.php/jast
Vol. 1 | No. 4 | Page 346 – 357 | 2023 |
ISSN : 2958-9568
Doi:10.59628/jast.v1i4.583

Characteristics of magnetohydrodynamic waves propagating in a magnetized relativistic degenerate plasma

Yusra A. A. Hager[1,*], Mahmood A. H. Khaled[2], and Mohamed A. Shukri[2]
[1]Departmentof physics, Faculty of Education, Sana'a University, Sana'a, Yemen.
[2]Department of physics, Faculty of Science, Sana'a University,Sana'a, Yemen.
Corresponding author:hager22013@gmail.com*

ARTICLE INFO
Article history:
Received: Aug24, 2023
Accepted: Oct17, 2023
Published: Nov, 2023

KEYWORDS
1. Magnetohydrodynamic waves.
2. Relativistic degenerate plasma.
3. Alfven waves.
4. Magnetosonic waves.
5. Quantam magnetohydrodynamic model.

ABSTRACT
A linear analysis was performed to investigate the basic properties of waves propagating in different directions in magnetized quantam plasma consisting of classical warm ions and relativistic degenerate electrons. We employ a quantum magnetohydrodynamic approach considering quantum corrections, spin effects in addition to relativity, and a polytropic index to obtain a generalized dispersion relation of the regarded plasma system. The considered system supports three types of magnetohydrodynamic waves: Alfven, fast, and slow magnetosonic waves. The obtained modified Alfven and magnetosonic dispersion relations are affected by the obliqueness factor, spin magnetization, and plasma beta β.The fast and slow modes were also altered owing to the Bohm potential and relativistic degenerate pressure impacts. Examination of the influences of these parameters revealed a significant modification of the phase velocity features of the magnetohydrodynamic waves. The results obtained may be applicable to relativistic magnetized quantum plasmas in astrophysical environments.

CONTENTS

1. Introduction:

Last decades have experienced tremendous growing in quantum plasma researches characterized by high density and low temperature, which are acquired considering quantum influences. In this sense, low-frequency with large-scale approximation waves known as magnetohydrodynamics (MHD) waves, basically Alfven and magnetosonic waves, can be produced. Alfven waves are electromagnetic oscillations that propagate parallel to the ambient magnetic field, whereas magnetosonic waves can propagate in either perpendicular or oblique directions [1]. They play crucial roles in both natural and laboratory plasmas and have significant applications in energy transport and dissipation in different environments, such as space and astrophysical plasmas, solar winds, and in earth-bound effects [2,3]; for example, Alfven waves are responsible for the heating of the solar corona. Furthermore, Alfven waves can play a major role in thermonuclear fusion plasmas, such as the International Tokamak Experimental Reactor (ITER) [4].

The magnetohydrodynamic (MHD) model was first used [5] for the derivation of Alfven waves in a magnetized plasma. Thereafter, the quantum magnetohydrodynamic (QMHD) approach was introduced [6] to investigate waves generated in low-temperature degenerate plasma considering quantum effects. Alfven and magnetosonicmode properties have been reported in various quantum plasma systems by applying the QMHD theory. For example, Marklund and Eliasson studied magnetosonic solitons, considering the effects of the Bohm potential and electron spin [7]. Alfven soliton properties were investigated by Brodin and Marklund by considering the collective spin effects in magnetized pair plasma [8]. Moreover, two-dimensional obliquely propagating magnetosonic waves have been considered inmulti- [9,10] and two-component [11-13] quantum plasmas, considering obliqueness effects [9-13] and quantum diffraction [19-22] with the incorporation of electron spin influences [12,13]. Misra and Ghosh [14] studied fast magnetosonic shock-like waves in dissipative quantum plasma with quantum tunneling and spin-alignment effects. Mushtaq and Vladimirov [15] described fast magnetosonic propagation in a linear regime and studied their properties in a large amplitude limit with the influence of diffraction, quantum statistics, and spin effects. Sahu et al. [16] analyzed the features of magnetosonic shock wave propagation in both small and arbitrary limits in the presence of magnetic diffusivity, Zeeman energy, and quantum diffraction impacts. Recently, Rahim et al. [17] investigated magnetosonic waves in separate spin-up and spin-down electron quantum plasmas based on the effects of quantum diffraction, dissipation, and spin polarization and traced their role in generating monotonic and oscillatory shock waves. Very recently, Hager et al. [18] examined the properties of fast and slow magnetohydrodynamic waves by considering the cumulative effects of the Coriolis force, dissipation influence, and quantum corrections represented by spin magnetization, quantum tunneling, and degeneracy forces. They explored the shock profile structures in a two-component plasma immersed in a uniform one-dimensional magnetic field.

On the other hand, the electron speed may become relatively high and can approach the speed of light in vacuum in degenerate plasma environments such as the interiors of white dwarf stars and magnetars [19], in addition to some practical situations such as intense laser–solid interaction experiments. In these cases, relativistic and quantum mechanical effects must be considered in a combined manner. Many authors have proposed various quantum plasma systems that consider degenerate relativistic impacts to investigate the features of electrostatic and electromagnetic waves in either unmagnetized [20-24] or magnetized [25-31] relativistically degenerate quantum plasmas. Zhenni et al. [25] investigated the propagation of electron acoustic waves by considering the tunneling effect in addition to the relativistic degenerate effect in magnetized quantum plasma. Abdikian and Mahmood [27] studied the characteristics of acoustic solitons in a three-component magnetized quantum plasma considering quantum corrections and relativistic degenerate pressure using a quantum hydrodynamic (QHD) model. QHD equations were considered to investigate solitary waves propagating obliquely in magnetized quantum plasma with relativistic degenerate electrons and positrons in addition to Bohm potential influences [30]. Recently, Chen et al. [31] studied high-frequency surface waves using the QHD model, considering the influences of the Bohm potential, quantum statistical pressure with relativistic degenerate effects, and exchange correlation impacts in spin-1/2 quantum plasmas.

To the best of our knowledge, there has not been any research on combining the relativistic degenerate effects and quantum corrections in addition to the obliqueness impacts in spin-1/2 quantum magneto-plasmas to study magnetohydrodynamic waves based on the QMHD theory. In this study, we explore the characteristics of magnetohydrodynamic waves in a magnetic quantum plasma propagating either oblique, across, or along the magnetic field direction, taking the quantum tunneling and spin effects with relativistic degenerate electrons into account. The remainder of this paper is organized as follows. We introduce the basic set of system equations, including quantum effects, magnetization, and relativistic degenerate impacts, and develop the one-fluid QMHD equations in Section 2. Section.3 is devoted to the formulation of the dispersion relation, and further analytical and numerical discussions on linear waves propagating in different directions are presented in Section 3. Finally, we summarize our results in Section 4.

1. Theoretical model and QMHD equations

We considered a quantum plasma system consisting of a mixture of relativistic degenerate electrons and classical warm ions. Such quantum plasmas are assumed to be embedded in a constant background magnetic field lying in the $x-z$ plane $\boldsymbol{B_0}$. The quantum contributions of ions are neglected owing to their large masses compared to those of electrons. Here, we assume that the ion pressure obeys the law $P_i = P_{i0}(n_i/n_{i0})^{\gamma}$ where n_i is the ion number density, n_{i0} is its equilibrium density, γ is the ratio of the specific heat, and $P_{i0} = n_{i0}k_B T_i$ with k_B is the Boltzmann constant and T_i is the ion temperature. For a relativistic degenerate electron, the following equation of state is defined [32]:

$$P_e = \frac{m_e^4 c^5}{24\,\pi^2\hbar^3}\left[R\,(2R^2-3)(1+R^2)^{1/2} + 3\sinh^{-1}(R)\right], \tag{1}$$

where c is the speed of light in vacuum, m_e is the electron mass. The quantity R is a dimensionless quantitythat measures the relativistic effect of electrons.

$$R = \frac{p_{Fe}}{m_e c} = R_0\left(\frac{n_e}{n_{e0}}\right)^{1/3},$$

in which $p_{Fe} = [3\,\pi^2\hbar^3(n_e/n_{e0})]^{1/3}$ is called the relativistic electronic Fermi momentum and $R_0 = (n_{e0}/n_c)^{1/3}$ is the relativistic degeneracy parameter, where $n_c = m_e^3 c^3/3\pi^2\hbar^3 \approx 5.9\times10^{29}\text{cm}^{-3}$ is a crucial density separates nonrelativistic and relativistic degeneracy regimes. In other words, the electrons become relativistically degenerate when their density $> 5.9\times10^{29}\text{cm}^{-3}$ or when $R_0 > 1$. It is easily noted that the gradient of the relativistic degenerate pressure can be written as

$$\boldsymbol{\nabla} P_e = n_e m_e c^2 \boldsymbol{\nabla}\sqrt{1+R^2}. \tag{2}$$

In this work, the relativistic degenerate electron is also under quantum effects, that is, the spin force and Bohm potential. Based on the above considerations, the dynamic equations of such quantum plasmas are given by

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla}.(n_i\mathbf{u}_i) = 0, \tag{3}$$

$$n m_i \frac{D\mathbf{u_i}}{Dt} = e n_i(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \gamma k_B T_i\left(\frac{n_i}{n_0}\right)^{\gamma-1}\boldsymbol{\nabla} n_i - \mathbf{R}_{ei}, \tag{4}$$

$$0 = -n_e e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \boldsymbol{\nabla} P_e + \mathbf{F}_Q + \mathbf{R}_{ei}\,, \tag{5}$$

where $\mathbf{u}_i(\mathbf{u}_e)$ and $n_i(n_e)$ are the vector velocity and number density of ions (electrons), respectively; γ is the ratio of specific heat;e is the electron charge; and $\mathbf{E}$ is the electric field vector. Here, $D/Dt = \partial/\partial t + (\boldsymbol{u_i}.\boldsymbol{\nabla})$ is the hydrodynamic derivative and $\mathbf{F}_Q$ represents the quantum force on electron, can be expressed as [33,34]

$$\mathbf{F}_Q = \frac{\hbar^2 n_e}{2m_e}\boldsymbol{\nabla}\left[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right] + n_e\mu_B\tanh\left(\frac{\mu_B B}{\varepsilon_{Fe}}\right)\boldsymbol{\nabla} B, \tag{6}$$

where first term is the Bohm potential gradient force due to the quantum tunneling effect while the second one represents the spin magnetization force in degenerate plasmas, $\mu_B = e\hbar/2m_e$ denotes the Bohr magneton and $B = |\mathbf{B}|$. The $\tanh(\mu_B B/\varepsilon_{Fe})$ function is the Langevin function owing to the magnetization of a spin 1/2 electron, $\mathcal{E}_{Fe} = \left[(3\,\pi^2 n_{e0})^{2/3}\hbar^2/2m_e\right] = (1/2)m_e c^2 R_0^2$ is the relativistic Fermi energy. The quantity $\mathbf{R}_{ei} = n\,m_e \nu_{ei}(\mathbf{u}_i - \mathbf{u}_e) = e n_0 \eta\,\mathbf{J}_p$ [35] is the rate of the transfer of momentum from ions to electrons by collisions with the current density $\mathbf{J_p}$ and the plasma resistivity $\eta = m_e \nu_{ei}/e^2 n_0$ where ν_{ei} is the electron-ion collisional frequency. The quasi-neutrality condition of this system is $n_e \approx n_i = n$. Hence, the current density becomes:

$$\mathbf{J_p} = en_i\mathbf{u_i} - en_e\mathbf{u_e} \approx en(\mathbf{u_i} - \mathbf{u_e}). \quad (7)$$

The relevant Maxwell equationsare

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J_p} + \mathbf{J_m}), \quad (8)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (9)$$

$$\nabla \cdot \mathbf{B} = 0\,, \quad (10)$$

where μ_0 is the magnetic permeability of free space and $\mathbf{J_m} = \nabla \times \mathbf{M}$ is the electron spin magnetization current density, $\mathbf{M}$ [33] is the mean magnetization:

$$\mathbf{M} = n_e\mu_B \tanh\left(\frac{\mu_B B}{\varepsilon_{Fe}}\right)\hat{\boldsymbol{b}}, \quad (11)$$

where $\hat{\boldsymbol{b}} = \mathbf{B}/B$ is the unit vector in the direction of $\mathbf{B}$. The displacement current in Eq.(8) was neglected because of its small value in the conducting medium compared to the total current density.

To derive the basic governing equations of the QMHD model, we substitute $\mathbf{u}_e$ from Eq.(7) into Eq.(5), we can express the electrical field in the form:

$$\mathbf{E} = -\mathbf{u_i} \times \mathbf{B} + \frac{1}{en}\big(\boldsymbol{J_p} \times \mathbf{B} - \nabla P_{Fe} + \mathbf{F_Q} + \mathbf{R_{ei}}\big). \quad (12)$$

By eliminating $\mathbf{E}$ from Eqs. (12) and (4) then, substituting from Eq.(8) to eliminate $\mathbf{J_p}$, we obtain:

$$nm_i\frac{D\mathbf{u_i}}{Dt} = \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} - (\nabla \times \mathbf{M}) \times \mathbf{B} - \nabla P_{Fe} + \mathbf{F_Q} - \gamma k_B T_i \left(\frac{n}{n_0}\right)^{\gamma-1} \nabla n. \quad (13)$$

By eliminating again $\mathbf{E}$ between Eqs.(9) and (12), and using Eq.(8), we obtain the magnetic induction equation as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u_i} \times \mathbf{B}) - \frac{\eta}{\mu_0}[\nabla \times (\nabla \times \mathbf{B})]. \quad (14)$$

Because the MHD wave frequency was assumed to be much smaller than the ion gyrofrequency, the Hall effect was neglected in this study. Now, using the approximation $\tanh(\mu_B B/\varepsilon_{Fe}) \approx \mu_B B/\varepsilon_{Fe}$, which is a valid approximation in most dense plasma systems where $\mu_B B \ll \varepsilon_{Fe}$, the basic set of one-fluid QMHD equations can be written as

$$\frac{\partial n}{\partial t} + \nabla.(n\mathbf{u_i}) = 0, \quad (15)$$

$$\frac{D\mathbf{u_i}}{Dt} = \frac{1}{\mu_0 n m_i}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{1}{nm_i}(\nabla \times \mathbf{M}) \times \mathbf{B} + \frac{1}{m_i}\left(\frac{\mu_B^2 B}{\varepsilon_{Fe}}\right)\nabla B + \frac{\hbar^2}{2m_e m_i}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) - \frac{1}{nm_i}\nabla P_{Fe} - \frac{1}{n}c_{Ti}^2\left(\frac{n}{n_0}\right)^{\gamma-1}\nabla n\,, \quad (16)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u_i} \times \mathbf{B}) + \eta_0 \nabla^2 \mathbf{B}, \quad (17)$$

where $\eta_0 = \eta/\mu_0$ and $c_{Ti} = \sqrt{\gamma k_B T_i/m_i}$ is the nondegenerate ion thermal speed. Here, the vector identities $\nabla \times (\nabla \times \mathbf{B}) = \nabla \cdot (\nabla \cdot \mathbf{B}) - \nabla^2\mathbf{B}$, with $(\nabla \cdot \mathbf{B}) = 0$ are used.

3. Generalized dispersion relation

In order to investigate the linear features of QMHD waves in such relativistic degenerate plasma, we represent all variables (i.e.,n , P_e, $\mathbf{u_i}$, $\mathbf{M}$ and $\mathbf{B}$) as a sum of their equilibrium values (denoted ‘0’) and a small perturbed component (denoted ‘1’) as follows:

$$\begin{pmatrix} n \\ P_e \\ \mathbf{u_i} \\ \mathbf{B} \\ \boldsymbol{M} \end{pmatrix} = \begin{pmatrix} n_0 \\ P_{e0} \\ \mathbf{0} \\ \mathbf{B_0} \\ \mathbf{M_0} \end{pmatrix} + \begin{pmatrix} n_1 \\ P_{e1} \\ \mathbf{u_1} \\ \mathbf{B_1} \\ \mathbf{M_1} \end{pmatrix}, \quad (18)$$

where ∇P_{e1} can be obtained from Eq. (2) to be

$$\nabla P_{e1} = \frac{m_e c^2 R_0^2}{3\sqrt{1+R_0^2}}\nabla n_1. \quad (19)$$

Substituting the expressions (18) and (19) into the QMHD equations (15)– (17), and after linearizing, we obtain

$$\frac{\partial n_1}{\partial t} + n_0(\nabla.\boldsymbol{u_1}) = 0, \quad (20)$$

$$\frac{\partial \boldsymbol{u_1}}{\partial t} = -\frac{1}{\mu_0 n_0 m_i}[\mathbf{B_0} \times (\boldsymbol{\nabla} \times \mathbf{B_1})] + \frac{1}{n_0 m_i}[\boldsymbol{B_0} \times (\boldsymbol{\nabla} \times \boldsymbol{M_1})] - \frac{C_{\text{eff}}^2}{n_0}\boldsymbol{\nabla} n_1 + \frac{\mu_B \varepsilon_0}{m_i}\boldsymbol{\nabla} B_1 + \frac{\hbar^2}{4 m_e m_i}\boldsymbol{\nabla}(\nabla^2 n_1), \quad (21)$$

$$\frac{\partial \mathbf{B_1}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{u_1} \times \mathbf{B_0}) + \eta_0 \nabla^2 \mathbf{B_1}. \quad (22)$$

Here, $\varepsilon_0 = \mu_B B_0 / \mathcal{E}_{Fe}$ is the normalized Fermi-Zeeman energy and C_{eff} is the effective acoustic speed modified by the relativistic degenerate effects as

$$C_{\text{eff}} = \left(\frac{c_{qs}^2}{3\sqrt{1+R_0^2}} + c_{Ti}^2\right)^{1/2}, \quad (23)$$

where $c_{qs} = R_0\sqrt{m_e c^2 / m_i}$ denotes the relativistic quantum ion acoustic speed. In a one-dimensional Cartesian coordinates, $\boldsymbol{\nabla} = (0,0,\partial/\partial z)$, consider the wave propagation is directed along a z-axis, so $\mathbf{k} = k\boldsymbol{e_z}$, and the geometry of the external uniform magnetic field is supposed to be in the plane $x - z$, $\mathbf{B_0} = B_0 \sin\alpha\, \mathbf{e_x} + B_0 \cos\alpha\, \boldsymbol{e_z}$, where α is the angle between the magnetic field and the unit vector $\mathbf{e_z}$, B_0 is the constant amplitude of the external magnetic field. Thus, the linear set of MHD equations (20) – (22) is

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial u_{z1}}{\partial z} = 0, \quad (24)$$

$$\frac{\partial u_{x1}}{\partial t} - \cos\alpha\left(\frac{V_A^2}{B_0} - \frac{\mu_B \varepsilon_0}{m_i}\right)\frac{\partial B_{x1}}{\partial z} + \frac{\mu_B B_0 \varepsilon_0}{\mathrm{n}_0 m_i}\cos\alpha \sin\alpha \frac{\partial n_1}{\partial z} = 0, \quad (25)$$

$$\frac{\partial u_{y1}}{\partial t} - \frac{\cos\alpha}{B_0}\left(V_A^2 - \frac{\mu_B B_0 \varepsilon_0}{m_i}\right)\frac{\partial B_{y1}}{\partial z} = 0, \quad (26)$$

$$\frac{\partial u_{z1}}{\partial t} = -\frac{\sin\alpha}{B_0}\left(V_A^2 - \frac{\mu_B B_0 \varepsilon_0}{m_i}\right)\frac{\partial B_{x1}}{\partial z} + \mu_B \varepsilon_0 \frac{B_0 \sin^2\alpha}{n_0 m_i}\frac{\partial n_1}{\partial z} + \frac{\hbar^2}{4 m_e m_i}\frac{\partial^3 n_1}{\partial z^3} - \frac{1}{n_0} C_{\text{eff}}^2 \frac{\partial n_1}{\partial z} + \frac{\mu_B \varepsilon_0}{m_i}\cos\alpha \frac{\partial B_{z1}}{\partial z} \quad (27)$$

$$\frac{\partial B_{x1}}{\partial t} = -B_0 \sin\alpha \frac{\partial u_{z1}}{\partial z} + B_0 \cos\alpha \frac{\partial u_{x1}}{\partial z} + \eta_0 \frac{\partial^2 B_{x1}}{\partial z^2}, \quad (28)$$

$$\frac{\partial B_{y1}}{\partial t} = B_0 \cos\alpha \frac{\partial u_{y1}}{\partial z} + \eta_0 \frac{\partial^2 B_{y1}}{\partial z^2}, \quad (29)$$

$$\frac{\partial B_{z1}}{\partial t} - \eta_0 \frac{\partial^2 B_{z1}}{\partial z^2} = 0. \quad (30)$$

Assuming a plane wave solution, all perturbed quantities are proportional to $\exp i(kz - \omega t)$ which means $\partial/\partial t = -i\omega$ and $\partial/\partial z = ik$, then

$$\omega n_1 - k \mathrm{n}_0 u_{z1} = 0, \quad (31)$$

$$\omega\, u_{x1} + k V_A^2 \frac{\cos\alpha}{B_0}\left(1 - \frac{1}{2}\beta \varepsilon_0^2\right) B_{x1} - \frac{k}{2 n_0}\beta \varepsilon_0^2 V_A^2 \cos\alpha \sin\alpha\, n_1 = 0, \quad (32)$$

$$\omega\, u_{y1} + k V_A^2 \frac{\cos\alpha}{B_0}\left(1 - \frac{1}{2}\beta \varepsilon_0^2\right) B_{y1} = 0, \quad (33)$$

$$\omega\, u_{z1} - k V_A^2 \frac{\sin\alpha}{B_0}(1 - \beta \varepsilon_0^2) B_{x1} + \frac{k}{n_0}\left(\frac{\beta \varepsilon_0^2 V_A^2 \sin^2\alpha}{2} - \frac{\hbar^2 k^2}{4 m_e m_i} - C_{\text{eff}}^2\right) n_1 + k \frac{\beta \varepsilon_0^2 V_A^2}{2 B_0}\cos\alpha\, B_{z1} = 0, \quad (34)$$

$$(\omega + i\eta_0 k^2) B_{x1} - k B_0 \sin\alpha\, u_{z1} + k B_0 \cos\alpha\, u_{x1} = 0, \quad (35)$$

$$(\omega + i\eta_0 k^2) B_{y1} + k B_0 \cos\alpha\, u_{y1} = 0, \quad (36)$$

$$(\omega + i\eta_0 k^2) B_{z1} = 0, \quad (37)$$

$\beta = c_{qs}^2 / V_A^2$ is the plasma-beta, $V_A = B_0 / \sqrt{\mu_0 n_0 m_i}$ is the Alfven speed.

3.1 Alfven waves

Note that the set of equations (31)–(37) splits into two partial subsets. The first one is formed by equations (33) and (36), describing the y-components B_{y1} and u_{y1}. These two equations lead to the following dispersion relation:

$$\omega^2 - C_A^2 k^2 \cos^2\alpha + i\eta_0 k^2 \omega = 0. \quad (38)$$

This is a modified dispersion relation for Alfven waves, where C_A is the modified Alfven velocity:

$$C_A = V_A\sqrt{1-\frac{1}{2}\beta\varepsilon_0^2}. \qquad (39)$$

It should be noted here that the quantum effects on the Alfven velocity arise mainly due to the presence of the spin magnetization current in such quantum plasma. In the limiting case when $\beta \ll 1$, the modified Alfven speed C_A tends to a pure Alfven speed V_A where $\varepsilon_0 < 1$ is valid in dense plasmas. The imaginary part of the dispersion relation in Eq. (38) controls the damping or growth of the modified Alfven waves. To discuss how this part affects Alfven's waves, we separate the dispersion relation into its real and imaginary parts by letting $\omega = \omega_r + i\omega_i$, Using Equation (38), we obtain $\omega_r^2 - \eta_0 k^2\omega_i - \omega_i^2 - C_A^2 k^2\cos^2\alpha + i(2\omega_i + \eta_0 k^2)\omega_r = 0$. Because $\omega_r \neq 0$, the imaginary part is reduced to $\omega_i = -\eta_0 k^2/2$, which reflects the energy dissipation owing to the plasma resistivity. Thus, the real dispersion relation for the modified Alfven waves becomes

$$\omega_r^2 - C_A^2 k^2\cos^2\alpha + \frac{1}{4}\eta_0^2 k^4 = 0. \qquad (40)$$

This implies that the imaginary part of the dispersion relation (38) controls the damping of the modified Alfven waves. Now, we consider the ideal case ($\eta_0 \to 0$), which leads to the dispersion relation

$$\omega^2 - C_A^2 k^2\cos^2\alpha = 0. \qquad (41)$$

It is observed that there is no effect of the electron spin-1/2 force on the modified Alfven waves because the spin is aligned parallel to the external magnetic field and therefore does not couple with the perturbed magnetic field. Moreover, this mode is independent of the Bohm potential and relativistic degeneracy. This phase speed can be oblique to the external magnetic field owing to its proportion to the term $\cos\alpha$.

The phase velocity properties of the modified Alfven waves are shown in Fig. (1-2). Figure 1 illustrates the phase velocity of modified Alfven waves normalized by the light speed against the relativistic degeneracy parameter R_0 for different values of α . It is clear that the phase velocity decreases with

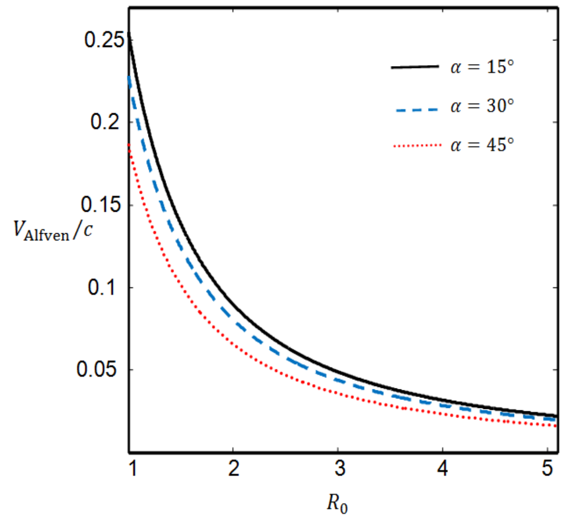

Figure. 1: Phase velocity of the modified Alfven waves versus the relativistic degeneracy parameter for different values of α where $B_0 = 2.79$ GT

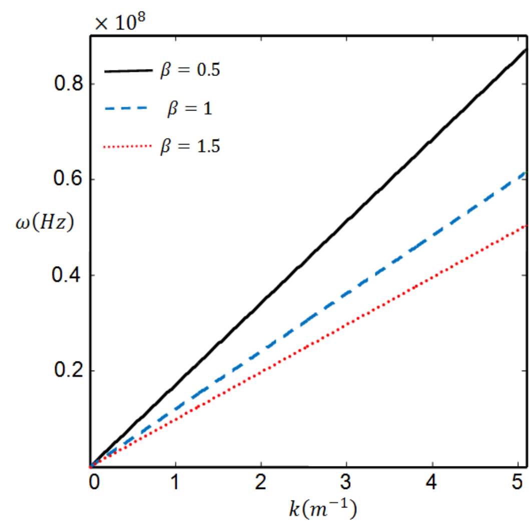

Figure. 2: Linear dispersion relation for different values of β where $\alpha = 30°, R_0 = 2$.

both R_0 and α, and the decay is faster for a weak relativistic degeneracy factor and slower for a larger R_0. The dispersion relation represented by the modified Alfven wave frequency versus the wave number shows linear behavior, which decreases as β parameter acquires higher values, as depicted in Fig.2.

3.2 Magnetosonic waves

The second partial set of equations is formed by equations (28), (29), (31), (32), and (34) and described the variables u_{x1} , u_{z1}, B_{x1} and n_1. The consistency condition yields the following:

$$u_{z1} = \frac{\omega}{kn_0} n_1, \tag{42}$$

$$u_{x1} = -\frac{k}{\omega}\frac{\cos\alpha}{B_0} V_A^2 \left(1 - \frac{1}{2}\beta\varepsilon_0^2\right) B_{x1} + \frac{k}{\omega}\frac{\beta\varepsilon_0^2 V_A^2}{2n_0}\cos\alpha\sin\alpha\, n_1, \tag{43}$$

$$B_{z1} = 0. \tag{44}$$

Substituting Eqs. (42-44) into (34) and (35) yields the following matrix:

$$\begin{pmatrix} \omega^2 - i\eta_0 k^2\omega - k^2 C_A^2\cos^2\alpha & -\frac{B_0\sin\alpha}{n_0}\left(\omega^2 - \frac{1}{2}k^2\beta\varepsilon_0^2 V_A^2\cos^2\alpha\right) \\ -n_0 k^2 V_A^2 \frac{\sin\alpha}{B_0}(1-\beta\varepsilon_0^2) & \left(\omega^2 + \frac{k^2\beta\varepsilon_0^2 V_A^2\sin^2\alpha}{2} - \frac{\hbar^2 k^4}{4m_e m_i} - k^2 C_{\text{eff}}^2\right) \end{pmatrix} \begin{pmatrix} B_{x1} \\ n_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

It should be noted that the determinant of this matrix must vanish, which leads to a general dispersion relation (45). It is clear that the general dispersion relation of the MHD waves (45) is considerably modified by the quantum Bohm potential (via $\hbar^2$) effect, electron spin-1/2 effect (via ε_0), and relativistic degenerate pressure effects (via C_{eff}).

$$\left[(\omega^2 - i\eta_0 k^2\omega - k^2 C_A^2\cos^2\alpha)\left(\omega^2 + \frac{1}{2}k^2\beta\varepsilon_0^2 V_A^2\sin^2\alpha - \frac{\hbar^2 k^4}{4m_e m_i} - k^2 C_{\text{eff}}^2\right)\right] - \left[\left(\omega^2 - \frac{1}{2}k^2\beta\varepsilon_0^2 V_A^2\cos^2\alpha\right)(1 - \beta\varepsilon_0^2)k^2 V_A^2\sin^2\alpha\right] = 0, \tag{45}$$

For the purpose of discussion, we consider the ideal case in which the plasma has perfect conductivity ($\eta_0 \to 0$), and the dispersion relation in (45) is reduced to (46)

$$(\omega^2 - k^2 C_A^2\cos^2\alpha)\left(\omega^2 + \frac{1}{2}k^2\beta\varepsilon_0^2 V_A^2\sin^2\alpha - \frac{\hbar^2 k^4}{4m_e m_i} - k^2 C_{\text{eff}}^2\right) - k^2 V_A^2(1-\beta\varepsilon_0^2)\left(\omega^2 - \frac{1}{2}k^2\beta\varepsilon_0^2 V_A^2\cos^2\alpha\right)\sin^2\alpha = 0, \tag{46}$$

To understand how the propagation modes are affected at different angles, we consider two limiting cases in Eq. (46) as:

Perpendicular propagation ($\alpha = \pi/2$)

When the propagation vector is perpendicular to the external magnetic field $\mathbf{B}_0$, the dispersion relation (46) takes the form:

$$\frac{\omega^2}{k^2} = V_A^2 + C_{\text{eff}}^2 + \frac{\hbar^2 k^2}{4m_e m_i} - \frac{3}{2}\beta\varepsilon_0^2 V_A^2, \tag{47}$$

In this case, only the compressional (fast) magnetosonic waves propagate perpendicularly (the slow mode cannot propagate $\perp \mathbf{B}_0$). Their propagation is affected by quantum corrections, in addition to the effects ofrelativistic electron degeneracy and warm ions, as shown in Eq. (47).

Parallel propagation ($\mathbf{B}_0 \parallel \mathbf{k}$)i.e., if $\alpha = 0$

In the limiting case, when the external magnetic field is in the same direction as the wave vector, the dispersion relation (46) is reduced to

$$(\omega^2 - k^2 C_A^2)\left(\omega^2 - k^2 C_{\text{eff}}^2 - \frac{\hbar^2 k^4}{4m_e m_i}\right) = 0, \tag{48}$$

It is noted from (48) that the fast wave becomes incompressible (the fast magnatosonic wave cannot propagate along the magnetic field) and degenerates into the modified Alfven wave.

$$\omega^2 - k^2 C_A^2 = 0, \tag{49}$$

In this case, there is a transverse wave along the magnetic field and the wave vector. The phase velocity is $V_p = C_A$. However, the dispersion relation (48) indicates the possibility of another wave mode. This is an ordinary quantum ion acoustic wave with a dispersion relationship.

$$\omega^2 = k^2\left(\frac{\hbar^2 k^2}{4m_e m_i} + C_{\text{eff}}^2\right), \tag{50}$$

As can be seen, no electric current density or magnetic field was associated with this wave. It is affected by quantum corrections ($\hbar$) in addition to the electron relativistic degeneracy parameter and the polytropic index of the ion. The phase velocity of this acoustic wave is.

$$V_{pa} = \sqrt{\frac{\hbar^2 k^2}{4m_e m_i} + C_{\text{eff}}^2}.$$

If we ignore the Bohm effect, $V_{pa} = C_{\text{eff}}$. Asimilar result was found by Rf. 34.

3.3 Arbitrary propagation for ideal MHD wave

In this case, the dispersion relation (46) can be written as

$$\omega^4 - \omega^2 k^2 A + k^4 B = 0, \tag{51}$$

where A and B are given by

$$A = C_{\text{eff}}^2 + V_A^2 + k^2\frac{\hbar^2}{4m_e m_i} - \left(\frac{1}{2} + \sin^2\alpha\right)\beta\varepsilon_0^2 V_A^2,$$

$$B = V_A^2\cos^2\alpha\left[\left(1 - \frac{1}{2}\beta\varepsilon_0^2\right)\left(\frac{\hbar^2 k^2}{4m_e m_i} + C_{\text{eff}}^2\right) - \frac{1}{4}V_A^2\beta^2\varepsilon_0^4\sin^2\alpha\right].$$

The solution of Eq. (51) corresponds to fast and slow magnetosonic waves and takes the form:

$$\frac{\omega^2}{k^2} = \frac{A}{2}\left(1 \pm \sqrt{1 - \frac{4B}{A^2}}\right), \tag{52}$$

where spectrum (52) represents the dispersion relation of oblique propagation, which has two distinct modes: the $(+)$ sign for the fast magnetosonic mode and the $(-)$ sign for the slow magnetosonic mode, which are affected by quantum corrections (via $\beta, \hbar$) and obliqueness influence (α) in addition to the ion polytropic index and electron relativistic degeneracy effects. Furthermore, the group velocity is:

$$V_g = \left(\frac{\hbar^2 k}{4m_e m_i}\right)\frac{\left(V_p^2 - V_A^2\cos^2\alpha\,(1 - \beta\varepsilon_0^2/2)\right)}{V_p\left(2V_p^2 + A\right)}. \tag{53}$$

As clear from Eq. (52), the dispersion relation of oblique propagation is modified by the effective speed C_{eff} . This speed appears to be affected by the ion polytropic index and electron relativistic degeneracy parameter R_0. The way in which the effective speed is changed according to the relativistic degeneracy parameter R_0 and consequently the wave velocities, is depicted in Fig. 3. Figure. 3 indicates that increasing the relativistic degeneracy parameter leads to an increase in the effective speed in a curved manner for lower R_0values and tends to be linear for higher values, while the warmness of ions via T_i has no noticeable influence (we obtain the same curve for different T_i values).

The phase velocity characteristics of the getting modes in the oblique direction, in addition to those propagating either parallel or perpendicular to the magnetic field direction,

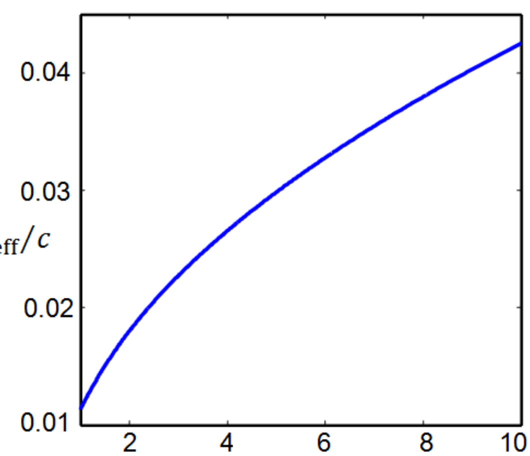

Figure. 3: Effective acoustic speed against relativistic degeneracy parameter R_0.

can be investigated graphically, as depicted in Figs. (4-8). Figures. 4 and 5 show a polar diagram of the phase velocity for the fast, slow and modified Alfven waves, in addition to the Alfven speed V_A.They clarified how the phase speed of each mode is affected by the propagation angle, relativistic degenercy parameter R_0 and β factor. It is obvious that the modified Alfven waves propagate along the magnetic fields$(\alpha = 0, \pi)$ with the maximum phase velocity and decrease as α increases. Their velocity equals zero in the transverse direction, that is, Alfven waves cannot propagate across the magnetic field while propagating with a faster velocity in the direction of the magnetic field. The phase velocity of the fast magnetosonic waves varies according to the propagation direction. It has the highest value for perpendicular propagation, which is described by Eq.(47), then they decrease slightly until they become incompressive and degenerate to a modified Alfven wave that travels at an Alfven speed along the magnetic field lines. Furthermore, slow magnetosonic waves behave like Alfven waves at lower speeds; they are both anisotropic. It is clear from these figures that asβ factor increases (i.e., the Alfven speed V_A decreases), the velocity of the slow mode increases, whereas the fast waves become faster because of the Alfven speed in the prependicular (maximum) and oblique directions. A comparison between Fig. 4 $(R_0 = 2.5)$ and Fig.5 $(R_0 = 1.2)$ reflects the influence of the relativistic degeneracy parameter R_0.The phase velocities of the fast and slow modes increased as R_0 decreased. Moreover, the fast mode, which can be considered approximately isotropic in Fig. 4, is no longer isotropic when$(R_0 = 1.2, \beta > 1)$ in Fig.5.

The polar plots in Figs. 4 and 5 generally show the phase velocity of the wave modes. For additional specifications, we explore the features of Alfven and magnetosonic waves against magnetic field angle α in Figs.6 and 7. Figure.6 displays the behavior of modified Alfven waves in addition to the fast and slow modes, where it can be seen that the modified Alfven waves and slow mode have a maximum velocity at $\alpha = 0$, and decreases as αincreases. They vanish at $\alpha = \pi/2$; whereas the fast wave velocity seems to be constant as α increases. Fast waves appear to have the same phase velocity over all α ranges. Figure. 7 exhibits the same waves for the same parameter unless the

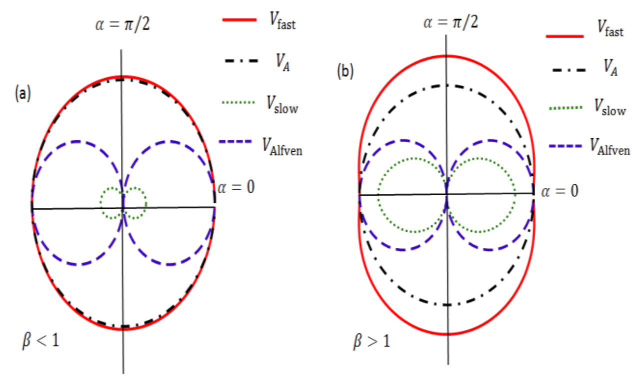

Figure. 4: Phase velocity polar diagram of fast, slow, modified Alfven waves and Alfven speed for different values of β where $R_0 = 2.5$, $T_i = 5000K$, $k = 1$.(a) $\beta = 0.5, B_0 = 3.45GT$(b) $\beta = 5, B_0 = 1.09GT$

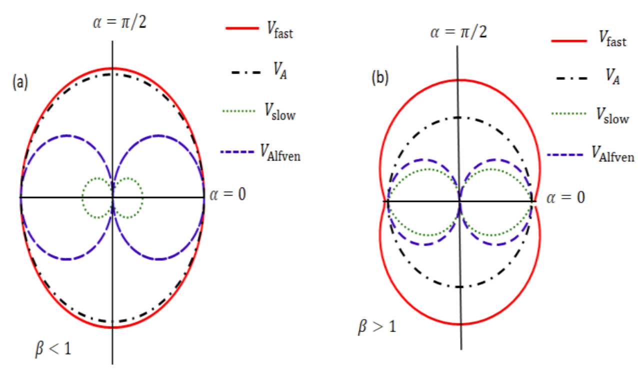

Figure. 5: Phase velocity polar diagram of fast, slow, modified Alfven waves and Alfven speed for different values of β where $R_0 = 1.2$, $T_i = 5000$ K, $k = 1$.(a) $\beta = 0.5, B_0 = 0.55GT$(b) $\beta = 5, B_0 = 0.174$ GT.

relativistic degeneracy parameter R_0. It is clear that corresponding to R_0 value $(R_0 = 1.2)$ the initial values of the fast and modified Alfven waves are lower than those in Fig. 6 $(R_0 = 2.5)$.The fast wave velocity V_{fast} increases as α increases, whereas the Alfven wave velocity decreases slowly compared to their behavior, as shown in Fig.6. Meanwhile, slow waves have the same initial velocity in both figures, but decrease faster as α increases, as shown in Fig.7.

The dependence of phase velocity on relativistic degeneracy parameter R_0 is shown in Fig. 8. It can be seen that the fast mode has larger values for small R_0values, then drops to a

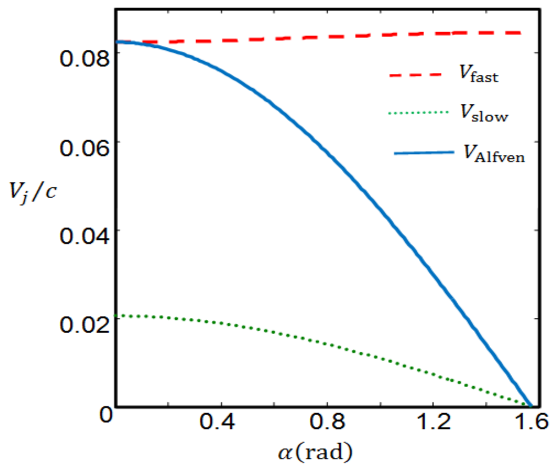

Figure. 6: Phase velocity of fast, slow and modified Alfven waves versus α where $R_0 = 2.5$, $T_i = 5000$℃, $k = 1$ and $\beta = 0.5$.

minimum value, after which it increases slightly in a linear manner. The modified Alfven wave velocity has large values for smaller R_0 values and decreases as R_0 values progress. Differently, the slow-mode velocity decreases slowly for small R_0 values, but the decrease tends to be linear as R_0 values increase until they propagate with the same velocity as the modified Alfven waves when ($R_0 > 6$).

4. Conclusion

In this work, the linear propagation of oblique magnetohydrodynamic waves in magnetized quantum plasma, including the effects of quantum diffraction, spin magnetization, ion polytropic pressure, and electron relativistic degenerate pressure, was studied extensively. We obtained a generalized dispersion relation by using the QMHD approach and linear analysis. The dispersion relation is reduced to three different equations according to the propagation direction described by the angle α.

Fast magnetosonic waves are produced for perpendicular propagation. Their propagation is modified by quantum corrections, in addition to the effects of relativistic electron degeneracy and warm ions. Parallel propagation introduces modified transverse Alfven waves that are altered by quantum spin magnetization and plasma beta β factors. In the case of arbitrary

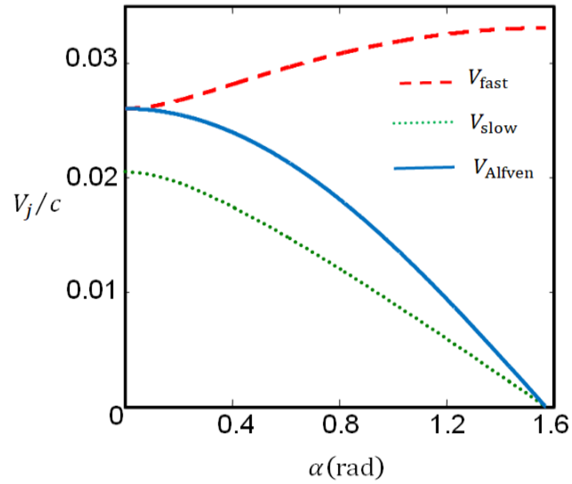

Figure. 7: Phase velocity of fast, slow and modified Alfven waves versus α where $R_0 = 2.5$, $T_i = 5000$℃, $k = 1$ and $\beta = 5$.

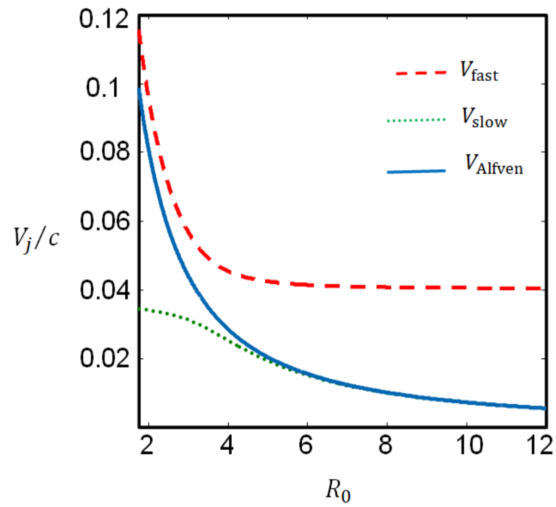

Figure. 8: Phase velocity of fast, slow and modified Alfven waves versus R_0 where $B_0 = 2.79\ GT$,$\alpha = 30°$, $T_i = 5000K$, $k = 1$.

propagation, it was found that fast and slow magnetosonic modes propagate under the influence of quantum corrections (via β,$\hbar$) and obliqueness impact (α) in addition to the ion warmness index and electron relativistic degeneracy effect $C_{\rm eff}$. Numerical analysis shows that fast modes propagate faster than modified Alfven waves do. However, the slow-mode phase velocity was lower than that of the modified Alfven wave. It was observed that the magnetohydrodynamic wave properties were overwhelmingly influenced by the relativistic degenerate parameter, plasma beta, and angle between the wave vector and magnetic field lines (α). The results of this work are general and can be applied to the study of oblique propagation of MHD waves in magnetized quantum plasma systems, and may be useful in understanding the energy transport mechanism in compact objects such as white dwarfs, where the influence of relativistic electron degeneracy becomes important.

* **Disclosures**:
The authors declare no conflict in interest.

5. References

[1] Cramer,N. F. *The Physics of Alfven Waves*, (Wiley VCH , Berlin ,2001), https://doi.org/10.1002/3527603123
[2] Ionson, J. A. Resonant electrodynamic heating of stellar coronal loops - an LRC circuit analog.*Astrophys. J.***1982**, 254, 318, https://doi.org/10.1086/159736
[3] Tamabechi, K.;Gilleland, J. R.;Sokolov,Y. A.; Toschi,R.; ITER Team.ITER conceptual design.*Nucl. Fusion***1991**, 31,1135, https://doi.org/10.1088/0029-5515/31/6/011
[4] Fasoli, A.;Gormenzano,C.; Berk, H. L.;Breizman, B. N.;Briguglio, S.;Darrow, D. S.;Gorelenkov,N. N.;Heidbrink, W. W.;Jaun, A;Konovalov, S. V.;Nazikian,R.; Noterdaeme, J.; Sharapov, S. E.;Shinohara, K.;Testa, D.;Tobita, K.; Todo, Y.;Vlad,G.;Zonca, F.Physics of energetic ions.*Nucl. Fusion***2007**,47, S264, https://doi.org/10.1088/0029-5515/47/6/s05
[5] Alfv'en, H. Existence of Electromagnetic-Hydrodynamic Waves.*Nature***1942**, 150, 405, https://doi.org/10.1038/150405d0
[6] Haas,F. A magnetohydrodynamic model for quantum plasmas.*Phys.Plasmas***2005**,12, 062117, https://doi.org/10.1063/1.1939947
[7] Marklund, M.;Eliasson,B.;Shukla, P. K. Magnetosonic solitons in a fermionic quantum plasma.*Phys. Rev. E***2007**, 76, 067401, https://doi.org/10.1103/PhysRevE.76.067401
[8] Brodin G.;Marklund, M. Spin solitons in magnetized pair plasmas.*Phys. Plasmas***2007**, 14, 112107, https://doi.org/10.1063/1.2793744
[9] Mushtaq,A.; Shah, H.A. Nonlinear Zakharov–Kuznetsov equation for obliquely propagating two-dimensional ion-acoustic solitary waves in a relativistic, rotating magnetized electron-positron-ion plasma.*Phys. Plasma***2005**, 12 , 072306, https://doi.org/10.1063/1.1946729
[10] Masood ,W.;Mushtaq, A. Obliquely propagating magnetosonic waves in multicomponent quantum magnetoplasma.*Phys. Lett. A***2008**, 372, 4283,https://doi.org/10.1016/j.physleta.2008.03.057
[11] Mushtaq, A.; Qamar, A. Parametric studies of nonlinear magnetosonic waves in two-dimensional quantum magnetoplasmas.*Phys. Plasmas***2009**, 16, 022301, https://doi.org/10.1063/1.3073669
[12] Mushtaq, A.; Vladimirov, S.V. Fast and slow magnetosonic waves in two-dimensional spin-1/2 quantum plasma.*Phys. Plasmas***2010**, 17, 102310, https://doi.org/10.1063/1.3493632
[13] Asenjo,F. A. The quantum effects of the spin and the Bohm potential in the oblique propagation of magnetosonic waves.*Phys. Lett. A***2012**,376, 2496, https://doi.org/10.1016/j.physleta.2012.06.023
[14] Misra,A. P.; Ghosh, N. K.Spin magnetosonic shock-like waves in quantum plasmas. *Phys. Lett. A***2008**, 372, 6412, https://doi.org/10.1016/j.physleta.2008.08.065
[15] Mushtaq,A.;Vladimirov, S.V. Arbitrary magnetosonicsolitary waves in spin 1/2 degenerate quantum plasma.*Eur. Phys. J. D***2011**, 64, 419, https://doi.org/10.1140/epjd/e2011-20374-x
[16] Sahu,B.; Choudhury, S.;Sinha,A. Small and arbitrary shock structures in spin 1/2 magnetohy-drodynamic quantum plasma.*Phys. Plasmas***2015**, 22, 022304, https://doi.org/10.1063/1.4907658
[17] Rahim,Z.;Adnan, M.;Qamar, A.Magnetosonic shock waves in magnetized quantum plasma with the evolution of spin-up and spin-down electrons.*Phys. Rev. E***2019**,100,053206, https://doi.org/10.1103/PhysRevE.100.053206
[18] Hager, Y.; Khaled, M.; Shukri, M.Magnetosonic waves propagation in a magnetorotating quantum plasma.*Phys. Rev. E***2023**,107,055202, https://doi.org/10.1103/PhysRevE.107.055202
[19] Shapiro,S. L.;Teukolsky,S. A. "*Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects*", (Wiley VCH, Weinheim, 2004), https://doi.org/10.1002/9783527617661
[20] Masood,W.; Eliasson, B. Electrostatic solitary waves in a quantum plasma with

relativistically degenerate electrons. *Phys. Plasmas***2011**, 18,03450, https://doi.org/10.1063/1.3556122

[21] Chandra, S.; Paul, S. N.; Ghosh, B. Electron-acoustic solitary waves in a relativistically degenerate quantum plasma with two-temperature electrons. .*Astrophys Space Sci***2013**, 343, 213, https://doi.org/10.1007/s10509-012-1097-3

[22] Rahman, A.; Kerr,M. Mc.; El-Taibany, W. F.; Kourakis,I.; Qamar,A. Amplitude modulation of quantum-ion-acoustic wavepackets in electron-positron-ion plasmas: Modulational instability, envelope modes, extreme waves.*Phys. Plasmas***2015**, 22, 02230, https://doi.org/10.1063/1.4907247

[23] Rahman,A.;Kourakis, I.;Qamar, A. Electrostatic Solitary Waves in Relativistic Degenerate Electron–Positron–Ion Plasma. *IEEE Trans. Plasma Sci*. **2015**, 43, 4, https://doi.org/10.1109/tps.2015.2404298

[24] Chandra, S. Propagation of Electrostatic Solitary Wave Structures in Dense Astrophysical Plasma : Effects of Relativistic Drifts & Relativistic Degeneracy Pressure. *Advances in Astrophysics***2016**, 1,3, https://doi.org/10.22606/adap.2016.13005

[25] Zhenni, Z.; Zhengwei, WU.; Chunhua, LI.; Weihong, Y. Electron Acoustic Solitary Waves in Magnetized Quantum Plasma with Relativistic Degenerated Electrons,*Plasma Sci. Technol.***2014** 16, 11 ,https://doi.org/10.1088/1009-0630/16/11/01

[26] Rehman, M. A.; Shah, H. A.; Masood, W.; Qureshi, M. N. S. Alfven solitary waves in nonrelativistic, relativistic, and ultra-relativistic degenerate quantum plasma.*Phys.Plasmas***2015**, 22, 102301, https://doi.org/10.1063/1.4932072

[27] Abdikian A.; Mahmood, S. Acoustic solitons in a magnetized quantum electron-positron-ion plasma with relativistic degenerate electrons and positrons pressure. *Phys. Plasmas***2016**, 23, 122303, https://doi.org/10.1063/1.4971447

[28] Abdikian, A.Modulational instability of ion-acoustic waves in magnetoplasma with pressure of relativistic electrons. *Phys. Plasmas***2017**, 24, 052123, https://doi.org/10.1063/1.4984247

[29] Ahmed , M. K.; Sah .O. P. Solitary kinetic Alfvén waves in denseplasmas with relativistic degenerate electrons and positrons, *Plasma Sci. Technol.***2019**, 21, 045301, https://doi.org/10.1088/2058-6272/aaf20f

[30] Soltani, H.; Mohsenpour, T.; Sohbatzadeh, F. Obliquely propagating quantum solitary waves in quantum-magnetized plasma with ultra-relativistic degenerate electrons and positrons.*Contribution plasma phys.***2019**, 59, e201900038, https://doi.org/10.1002/ctpp.201900038

[31] Chen, S.; Li, C.; Zha, X.; Zhang, X.; Xia, Z. High-frequency surface waves in quantum plasmas with electrons relativistic degenerate and exchange-correlation effects. *Chin. J. Phys***2020**,68,79, https://doi.org/10.1016/j.cjph.2020.09.013

[32] Chandrasekhar,S. *An introduction to the study of stellar structure*, (Dover Publications, New York, 1957).ISBN 13:9780486604138

[33] Kittel, C. *Introduction to Solid State Physics*, 8th ed. (John Wiley & Sons, New York, 2005).ISBN 13: 9780471415268

[34] Pathria,R. K. *Statistical Mechanics* (Butterworth-Heinemann, Oxford, 1996). ISBN 0 7506 2469 8

[35] Goedbloed, J. P.;Poedts, S. *Principles of Magnetohydrodynamics* (Cambridge University Press, Cambridge, England,2004). https://doi.org/10.1017/CBO9780511616945

[36] El-Labany, S. K.;El-Taibany, W. F.;El-Samahy, A. E.; Hafez, A. M.; Atteya, A.Ion Acoustic Solitary Waves in Degenerate Electron-Ion Plasmas.*IEEE Trans. Plasma Sci*.**2016**, 44, 842, https://doi.org/10.1109/tps.2016.2539258