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Quantum effects on cylindrical dust-ion acoustic waves in a semiclassical dense dusty Plasma

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ABSTRACT: The nonlinear cylindrical dust ion acoustic waves are studied in a collisional unmagnetized dense dusty plasma medium containing inertialess degenerated electrons and positrons, fluid ions, negatively charged dust fluid and neutrals in the background, including exchange-correlation effects of both electrons and positrons. Employing the reductive perturbation method, the damped Kadomstev–Petviashvili equation is derived in the framework of two-dimensional cylindrical geometry. An approximate analytical solution of this equation is also discussed. The quantum and geometrical effects on the dust ion acoustic waves have been investigated. It is found that the dust ion acoustic wave is modified by quantum diffraction, Fermi statistics and exchange-correlation potential. Moreover, it has been found that the nebulon structures of quantum dust ions acoustic wave are formed due to the Cartesian geometry and the transverse perturbation. It is also observed that the nebulon structure is significantly modified by the exchange-correlation effects.

CONTENTS

I. Introduction

It is well known that the dust ion acoustic waves (DIAWs) are ion acoustic waves modified by the presence of massive charged dust particles in ordinary electron-ion plasma. The DIAWs were investigated theoretically by Shukla and Silin [1] and experimentally confirmed in low-temperature dusty plasma [2, 3]. In the past years, a number of researchers have investigated the basic properties of DIAWs in different dusty plasma systems [4-10]. Most of those investigations were limited to the classical plasmas, which are generally characterized by low density and high temperature. However, when the plasma particles have a very low temperature and high-number density, the de Broglie wavelength of the plasma particle could be comparable to the dimension of the plasma system (i.e. the particle Debye length becomes smaller than the thermal de-Broglie wavelength). Such dense plasma behaves like a Fermi gas, so the quantum effects may play a central role in the behavior of plasma particles and in the properties of the wave modes [11,12]. Nowadays, dense plasma has become one of the most important areas of research in plasma physics due to its vital role in many fields of science such as dense astrophysical systems, [13] microelectronic devices [14] and laser plasmas [15].

Over the past years, many theoretical investigations have been conducted on propagation features of DIAWs in a quantum dusty plasma medium assuming stationary dust grains [16-21]. However, this assumption may be a good approximation when the dust grains are heavy and large in comparison to the ions, but it is not a general case. In fact, the dust grains are existed in most environments of space and astrophysical dusty plasmas from smaller to larger size [21] where the mass ratio between dust grain and ion lies in the range 10^3-10^4 and the number of charges residing on the dust grain is also in the range 10^3-10^4 as required. In such plasmas, the dynamics of dust grains may be taken into account along with ions dynamics. Accordingly, several authors have investigated the propagation characteristics of quantum DIAWs in dusty plasmas, including the dynamics of dust grains as well as ions [21-25]. For example, Emadi and Zahed [24] have investigated the linear and nonlinear properties of DIAWs in a magnetized multi-component quantum dusty plasma composed of inerialess degenerate electrons and positrons, inertial cold ions and negatively charged dust particles. They are considering the effects of quantum diffraction and quantum statistics. Mushtaq et al. [25] have studied the damped DIAWs in a collisional, unmagnetized quantum dusty plasma which contains degenerate electrons, ions, neutrals and negatively charge dust grains, including quantum electron exchange-correlation effect. Most of these works [21-25] were based on the one-dimensional Korteweg–deVries equation. Ali et al. [26] have examined the nonlinear characteristics of the two-dimensional (2D) cylindrical quantum DIAWs considering the effects of quantum statistics and quantum Bohm potential in a collisionless unmagnetized dusty plasma. They found that the quantum DIAWs is significantly affected by cylindrical geometry. Recently, Khaled et al. [27] have investigated the nonlinear propagation of ion acoustic waves in a quantum electron-positron-ion plasma, in the framework of Zakarov-Kuznetsov equation, including the influence of electron/positron exchange-correlation effect. To the best of our knowledge there is no detailed investigation about the propagation of DIAWs in a 2D cylindrical quantum dusty plasma consisting of inertialess degenerate electrons and positrons, inertial ions and negatively charged dust particles, including quantum electron/positron exchange-correlation effects. Therefore, the aim of this paper is to examine the nonlinear structures of cylindrical DIAWs in a collisional unmagnetized quantum dusty plasma medium consisting of degenerate electrons and positrons, mobile ions and negatively charged dust grains as well as neutrals in the background, and then we examine the influence of quantum parameters and non-planar geometry on the nonlinear structures of the quantum DIAWs in such dusty plasma model.

This paper is organized as follows. Sect. 2 provides the basic set of equations for cylindrical DIAWs in a dense dusty plasma under consideration. The nonlinear damped cylindrical Kadomstev–Petviashvili (DCKP) equation in 2D cylindrical geometry is derived in Sec. 3. The solitary wave solution of the DCKP equation is obtained in Sect. 4. Numerical results and discussion are provided in Sect. 5. The conclusion is presented in Sect.6.

2. Mathematical Model

We consider an unmagnetized collisional multi-component dense dusty plasma system consisting of inertialess degenerated electrons, inertialess degenerated positrons, inertial ions and negatively charged dust grains, in the presence of neutrals in the background. We assume a collision effect between dust grains and neutrals while the collisions of ions or electrons/positrons with neutrals are neglected due to lighter mass. The quantum behavior of both dust grains and ions is neglected due to their heavy mass. Accordingly, the governing equations of DIAWs, including the dynamics of dust grains as well as ions dynamics are given by

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \boldsymbol{u}_j) = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}_i}{\partial t} + (\boldsymbol{u}_i \cdot \nabla)\boldsymbol{u}_i = -\frac{e}{m_i}\nabla\varphi, \tag{2}$$

$$\frac{\partial \boldsymbol{u}_d}{\partial t} + (\boldsymbol{u}_d \cdot \nabla)\boldsymbol{u}_d = \frac{Z_d e}{m_d}\nabla\varphi - \nu_{dn}\boldsymbol{u}_d, \tag{3}$$

$$\nabla^2\varphi = \frac{e}{\epsilon_0}(Z_d n_d + n_e - n_p - n_i), \tag{4}$$

where n_j, $\boldsymbol{u}_j$, and m_j are the number density, fluid velocity and mass of ions ($j = i$) and dust grains ($j = d$). φ is electrostatic potential, Z_d is the number of charges residing on the dust grain, e is the electronic charge, ν_{dn} is the collisional frequency of dust-neutrals and $n_e(n_p)$ represents the number density of degenerated electrons (positrons). We consider the quantum effects of electrons and positrons are encountered in terms of Fermi pressure, Bohm potential and exchange-correlation potential. Therefore, the inertialess momentum equation for electrons/positrons is given by

$$\pm e\nabla\varphi - \frac{\nabla P_{Fs}}{n_s} + \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n_s}}{\sqrt{n_s}}\right) - \nabla V_s^{xc} = 0, \tag{5}$$

where the sign (+) with electrons while and the sign (–) for positrons, $m = m_e = m_p$ is the mass of electron or positron and $\hbar$ is the Planck constant divided by 2π. Here, P_{Fs} is the Fermi pressure of plasma species s ($s = e$ for electrons and $s = p$ for positrons). In a 2D Fermi gas, $P_{Fs} = E_{Fs}n_s^2/2n_{s0,}$, where n_{s0} is the equilibrium number density of plasma species s and $E_{Fs}(= k_B T_{Fs})$ is the Fermi energy, in which $T_{Fs} = \left(2\pi^2 n_{s0,}\right)^{2/3}\hbar^2/2mk_B$ represents the Fermi temperature and k_B is the Boltzmann constant. The equilibrium number densities of electrons n_{e0}, positrons n_{p0}, ions n_{i0}, and dust grains n_{d0} are related by the charge neutrality condition as: $Z_d n_{d0} + n_{e0} = n_{p0} + n_{i0}$. In Eq. (5), V_s^{xc}represents the exchange-correlation potential which is given by [25, 27]

$$V_s^{xc} = -\left(\frac{0.985e^2}{4\pi\epsilon_0}\right)n_s^{1/3}\left[1 + \frac{0.034}{a_B n_s^{1/3}}\ln\left(1 + 18.367a_B n_s^{1/3}\right)\right], \tag{6}$$

where $a_B = 4\pi\epsilon_0\hbar^2/me^2$ is the Bohr radius and ϵ_0 is the permittivity of free space. In the dense plasmas, Eq. (6) can be simplified to [25, 27]

$$V_s^{xc} = -\left(\frac{1.6e^2}{4\pi\epsilon_0}\right)n_s^{1/3} + \left(\frac{5.62\hbar^2}{m}\right)n_s^{2/3}. \tag{7}$$

Now, we introduce the following dimensionless variables:

$$N_d = \frac{n_d}{n_{d0}}, \quad N_i = \frac{n_i}{n_{i0}}, \quad N_e = \frac{n_e}{n_{e0}}, \quad N_p = \frac{n_p}{n_{p0}}, \quad \boldsymbol{V}_{d,i} = \frac{\boldsymbol{u}_{d,i}}{C_i}, \quad \phi = \frac{e\varphi}{2E_{Fs}}, \quad \nabla \to \frac{C_i}{\omega_{pi}}\nabla, \quad t \to \omega_{pi}t, \tag{8}$$

where $\omega_{pi} = (e^2 n_{i0}/\epsilon_0 m_i)^{1/2}$ is the ion plasma frequency and $C_i = (2E_{Fe}/m_i)^{1/2}$ is the ion-sound speed. Introducing the dimensionless variables (8) into Eqs. (1)-(7), the normalized basic equations of DIAWs can be written in 2D cylindrical coordinates as:

$$\frac{\partial N_d}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rN_d V_{dr}) + \frac{1}{r}\frac{\partial}{\partial\theta}(N_d V_{d\theta}) = 0, \tag{9}$$

$$\frac{\partial V_{dr}}{\partial t} + V_{dr}\frac{\partial V_{dr}}{\partial r} + \frac{V_{d\theta}}{r}\frac{\partial V_{dr}}{\partial\theta} - \frac{V_{d\theta}^2}{r} - \mu_d\frac{\partial\phi}{\partial r} + \nu V_{dr} = 0, \tag{10}$$

$$\frac{\partial V_{d\theta}}{\partial t} + V_{dr}\frac{\partial V_{dr}}{\partial r} + \frac{V_{d\theta}}{r}\frac{\partial V_{dr}}{\partial\theta} + \frac{V_{dr}V_{d\theta}}{r} - \mu_d\frac{1}{r}\frac{\partial\phi}{\partial\theta} + \nu V_{d\theta} = 0, \tag{11}$$

$$\frac{\partial N_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rN_i V_{ir}) + \frac{1}{r}\frac{\partial}{\partial\theta}(N_i V_{i\theta}) = 0, \tag{12}$$

$$\frac{\partial V_{ir}}{\partial t} + V_{ir}\frac{\partial V_{ir}}{\partial r} + \frac{V_{i\theta}}{r}\frac{\partial V_{ir}}{\partial\theta} - \frac{V_{i\theta}^2}{r} + \frac{\partial\phi}{\partial r} = 0, \tag{13}$$

$$\frac{\partial V_{i\theta}}{\partial t} + V_{ir}\frac{\partial V_{ir}}{\partial r} + \frac{V_{i\theta}}{r}\frac{\partial V_{ir}}{\partial\theta} + \frac{V_{ir}V_{i\theta}}{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta} = 0, \tag{14}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = \beta_d N_d + \mu N_e - \mu p N_p - N_i, \tag{15}$$

and

$$\frac{H^2}{2}\frac{1}{\sqrt{N_e}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sqrt{N_e}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\sqrt{N_e}}{\partial\theta^2}\right] + \phi - \frac{1}{4}N_e + \alpha N_e^{1/3} - \gamma N_e^{2/3} = \alpha - \gamma - \frac{1}{4}, \tag{16}$$

$$\frac{H^2}{2}\frac{1}{\sqrt{N_p}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sqrt{N_p}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\sqrt{N_p}}{\partial\theta^2}\right] - \phi - \frac{\sigma}{4}N_p + p^{1/3}\alpha N_p^{1/3} - p^{2/3}\gamma N_p^{2/3} = p^{1/3}\alpha - p^{2/3}\gamma - \frac{\sigma}{4}, \tag{17}$$

where $\mu_d = Z_d m_i/m_d$ is the mass ratio of ion-to-dust, $\mu = n_{e0}/n_{i0}$ is the equilibrium density ratio of positron-to-ion, $p = n_{p0}/n_{e0}$ is the equilibrium density ratio of positron-to-electron density, $\beta_d = Z_d n_{d0}/n_{i0} = 1 - \mu(1-p)$ and $\nu = \nu_{dn}/\omega_{pi}$ is the normalized collisional frequency. In the Eqs. (16) and (17), the parameter $H = \omega_{pi}\hbar/\sqrt{mm_i C_i^2}$ is the normalized quantum parameter, $\sigma = T_{Fp}/T_{Fe}$ is the Fermi temperature ratio of positron-to-electron which related to the ratio p by $\sigma = p^{2/3}$. The parameters γ and α represent the exchange-correlation coefficients where $\gamma = 5.65\left(\hbar^2 n_{e0}^{2/3}/2mE_{Fe}\right) \approx 0.59$ and $\alpha = 1.62\left(e^2 n_{e0}^{1/3}/8\pi\epsilon_0 E_{Fe}\right)$.

3. Derivation of DCKP Equation

To obtain the nonlinear dynamical equation for 2D quantum DIAWs, we use the reductive perturbation technique (RPT) [28] in the 2D cylindrical geometry. Accordingly, the normalized independent variables (r,θ,t) are stretched as

$$\rho=\epsilon^{1/2}(r-w_0t),\qquad \vartheta=\epsilon^{-1/2}\theta,\qquad \tau=\epsilon^{3/2}t, \tag{18}$$

where ϵ is a smallness parameter measuring strength of nonlinearity and w_0 denotes the normalized phase velocity of quantum DIAWs (to be determined later on). The normalized dependent variables (N_s, N_j, V_{Jr}, ϕ) can be expanded in a power series of ϵ as

$$\begin{pmatrix}N_s\\N_j\\V_{Jr}\\\phi\end{pmatrix}=\begin{pmatrix}1\\1\\0\\0\end{pmatrix}+\epsilon\begin{pmatrix}N_s^{(1)}\\N_j^{(1)}\\V_{Jr}^{(1)}\\\phi^{(1)}\end{pmatrix}+\epsilon^2\begin{pmatrix}N_s^{(2)}\\N_j^{(1)}\\V_{Jr}^{(2)}\\\phi^{(2)}\end{pmatrix}+\cdots. \tag{19}$$

For $V_{J\theta}$ and ν we have taken:

$$V_{J\theta}=\epsilon^{3/2}V_{J\theta}^{(1)}+\epsilon^{5/2}V_{J\theta}^{(2)}+\cdots,\qquad \nu=\epsilon^{3/2}\nu_0. \tag{20}$$

Substituting Eqs. (18)-(20) into Eqs. (9)-(17), and collecting the terms of same powers of ϵ. For lowest order of ϵ, the following relations are obtained

$$N_d^{(1)}=-\frac{\mu_d}{w_0^2}\phi^{(1)},\qquad V_{dr}^{(1)}=-\frac{\mu_d}{w_0}\phi^{(1)}, \tag{21}$$

$$N_i^{(1)}=\frac{1}{w_0^2}\phi^{(1)},\qquad V_{ir}^{(1)}=\frac{1}{w_0}\phi^{(1)}, \tag{22}$$

$$N_e^{(1)}=\frac{6}{3-2\alpha_e}\phi^{(1)},\quad N_p^{(1)}=-\frac{6}{3\sigma-2\alpha_p}\phi^{(1)}, \tag{23}$$

where $\alpha_e=\alpha-2\gamma$, and $\alpha_p=p^{1/3}(\alpha-2p^{1/3}\gamma)$. The lowest order terms of θ-component of momentum equations of dust grains and ions are given by

$$\frac{\partial V_{d\theta}^{(1)}}{\partial\rho}=-\frac{\mu_d}{w_0^2\tau}\frac{\partial\phi^{(1)}}{\partial\vartheta}, \tag{24}$$

$$\frac{\partial V_{i\theta}^{(1)}}{\partial\rho}=\frac{1}{w_0^2\tau}\frac{\partial\phi^{(1)}}{\partial\vartheta}, \tag{25}$$

and the lowest order Poisson's equation yields:

$$\beta_dN_d^{(1)}+\mu N_e^{(1)}-\mu pN_p^{(1)}-N_i^{(1)}=0. \tag{26}$$

Equation (26) together with Eqs. (21)-(23), give us the normalized phase velocity of quantum DIAWs as

$$w_0=\sqrt{\frac{(\beta_d\mu_d+1)}{6\mu}}\left(\frac{1}{3-2\alpha_e}+\frac{p}{3\sigma-2\alpha_p}\right)^{-1/2}, \tag{27}$$

Clearly, the w_0 is modified by the quantum statistical and exchange-correlation effects (via the dimensionless parameters p,σ , α_p and α_e).

The next-order in ϵ gives the following set of coupled equations

$$\frac{\partial N_d^{(1)}}{\partial\tau}-w_0\frac{\partial N_d^{(2)}}{\partial\rho}+\frac{\partial V_{dr}^{(2)}}{\partial\rho}+\frac{\partial N_{dr}^{(1)}V_{dr}^{(1)}}{\partial\rho}+\frac{V_{dr}^{(1)}}{w_0\tau}+\frac{1}{w_0\tau}\frac{\partial V_{d\theta}^{(1)}}{\partial\vartheta}=0, \tag{28}$$

$$\frac{\partial V_{dr}^{(1)}}{\partial\tau}-w_0\frac{\partial V_{dr}^{(2)}}{\partial\rho}+V_{dr}^{(1)}\frac{\partial V_{dr}^{(1)}}{\partial\rho}-\mu_d\frac{\partial\phi^{(2)}}{\partial\rho}+\nu_0V_{dr}^{(1)}=0, \tag{29}$$

$$\frac{\partial N_i^{(1)}}{\partial\tau}-w_0\frac{\partial N_i^{(2)}}{\partial\rho}+\frac{\partial V_{ir}^{(2)}}{\partial\rho}+\frac{\partial N_{ir}^{(1)}V_{ir}^{(1)}}{\partial\rho}+\frac{V_{ir}^{(1)}}{w_0\tau}+\frac{1}{w_0\tau}\frac{\partial V_{i\theta}^{(1)}}{\partial\vartheta}=0, \tag{30}$$

$$\frac{\partial V_{ir}^{(1)}}{\partial\tau}-w_0\frac{\partial V_{ir}^{(2)}}{\partial\rho}+V_{ir}^{(1)}\frac{\partial V_{ir}^{(1)}}{\partial\rho}+\frac{\partial\phi^{(2)}}{\partial\rho}=0, \tag{31}$$

$$\frac{\partial^3\phi^{(1)}}{\partial\rho^3}=\mu\frac{\partial N_e^{(2)}}{\partial\rho}-\mu p\frac{\partial N_p^{(2)}}{\partial\rho}+\beta_d\frac{\partial N_d^{(2)}}{\partial\rho}-\frac{\partial N_i^{(2)}}{\partial\rho}, \tag{32}$$

and

$$N_e^{(2)}=\frac{6}{3-2\alpha_e}\phi^{(2)}-\frac{12(\alpha-\gamma)}{(3-2\alpha_e)^3}\left[\phi^{(1)}\right]^2+\frac{9H^2}{(3-2\alpha_e)^2}\frac{\partial^2\phi^{(1)}}{\partial\rho^2}, \tag{33}$$

$$N_p^{(2)}=-\frac{6}{3\sigma-2\alpha_p}\phi^{(2)}-\frac{12p^{1/3}\left(\alpha-p^{1/3}\gamma\right)}{\left(3\sigma-2\alpha_p\right)^3}\left[\phi^{(1)}\right]^2-\frac{9H^2}{\left(3\sigma-2\alpha_p\right)^2}\frac{\partial^2\phi^{(1)}}{\partial\rho^2}, \tag{34}$$

Substituting Eq. (21) into Eqs. (28) and (29) and eliminating the term of second-order perturbed i.e. $\partial V_{dr}^{(2)}/\partial\rho$, one can obtain the following equation

$$\frac{\partial N_d^{(2)}}{\partial \rho} = -\frac{2\mu_d}{w_0^3}\frac{\partial \phi^{(1)}}{\partial \tau} + 3\frac{\mu_d^2}{w_0^4}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \rho} - \frac{\mu_d}{w_0^3 \tau}\phi^{(1)} - \frac{\nu_0 \mu_d}{w_0^3}\phi^{(1)} - \frac{\mu_d}{w_0^2}\frac{\partial \phi^{(2)}}{\partial \rho} + \frac{1}{w_0^2 \tau}\frac{\partial V_{d\theta}^{(1)}}{\partial \vartheta}, \quad (35)$$

Likewise, Eqs. (22), (30) and (31) give

$$\frac{\partial N_i^{(2)}}{\partial \rho} = \frac{2}{w_0^3}\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{3}{w_0^4}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \rho} + \frac{\phi^{(1)}}{w_0^3 \tau} + \frac{1}{w_0^2}\frac{\partial \phi^{(2)}}{\partial \rho} + \frac{1}{w_0^2 \tau}\frac{\partial V_{i\theta}^{(1)}}{\partial \vartheta}, \quad (36)$$

Inserting Eqs. (33)-(36) into Eq. (32) and solving with aid of Eqs. (24) and (25), the following equation is obtained

$$\frac{\partial}{\partial \rho}\left(\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \rho} + B\frac{\partial^3 \varphi^{(1)}}{\partial \rho^3} + C\varphi^{(1)} + \frac{\phi^{(1)}}{2\tau}\right) + \frac{1}{2w_0 \tau^2}\frac{\partial^2 \phi^{(1)}}{\partial \vartheta^2} = 0. \quad (37)$$

Equation (37) is the DCKP equation, in which A is the nonlinear coefficient, B is the dispersive coefficient and C represents the coefficient of dissipation. The coefficients A, B and C are respectively given by

$$A = \frac{3(1-\beta_d \mu_d^2)}{2w_0(\beta_d \mu_d + 1)} + \frac{24\mu w_0^3}{(\beta_d \mu_d + 1)}\left[\frac{(\alpha-\gamma)}{(3-2\alpha_e)^3} - \frac{p^{4/3}(\alpha - p^{1/3}\gamma)}{(3\sigma - 2\alpha_p)^3}\right], \quad (38)$$

$$B = \frac{w_0^3}{2(\beta_d \mu_d + 1)}\left[1 - \frac{9\mu H^2}{(3-2\alpha_e)^2} - \frac{9\mu H^2 p}{(3\sigma - 2\alpha_p)^2}\right], \quad (39)$$

$$C = \frac{\nu_0}{2}\frac{\beta_d \mu_d}{\beta_d \mu_d + 1}. \quad (40)$$

As it is clear from (38), (39) and (40) that both the nonlinear and dispersion coefficients are modified by the exchange-correlation effects, while the damping coefficient (C) depends on the normalized collisional frequency ν_0, dust concentration β_d, and mass ratio μ_d. The effect of quantum diffraction appears only in the dispersion coefficient (B) through the quantum parameter H.

4. Solitary Wave Solutions of DCKP Equation

To get the solitary wave solution of Eq. (37), we introduce the following transformation [26]

$$\xi = \rho - \frac{1}{2}w_0 \vartheta^2 \tau, \quad \bar{\tau} = \tau. \quad (41)$$

Thus,

$$\phi^{(1)}(\rho, \vartheta, \tau) = \phi^{(1)}(\xi, \bar{\tau}). \quad (42)$$

Accordingly,

$$\frac{\partial}{\partial \rho} = \frac{\partial}{\partial \xi}, \quad \frac{\partial^3}{\partial \rho^3} = \frac{\partial^3}{\partial \xi^3}, \quad \frac{\partial}{\partial \tau} = \frac{\partial}{\partial \bar{\tau}} - \frac{1}{2}w_0 \vartheta^2 \frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial \vartheta^2} = w_0^2 \vartheta^2 \tau^2 \frac{\partial^2}{\partial \xi^2} - w_0 \tau \frac{\partial}{\partial \xi}. \quad (43)$$

Substituting (42) and (43) into Eq. (37), we obtain the following equation

$$\beta_d N_d^{(1)} + \mu N_e^{(1)} - \mu p N_p^{(1)} - N_i^{(1)} = 0. \quad (44)$$

which represents the Damped Korteweg-deVries (DKdV) equation in the ξ, $\bar{\tau}$ space. Obviously, when the collision effect is neglected, the damping term $C\phi^{(1)}$ can be removed and then Eq. (44) reduces to the well-known Korteweg-deVries (KdV) equation

$$\frac{\partial \phi^{(1)}}{\partial \bar{\tau}} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \quad (45)$$

which has the solitary wave solution [26]

$$\phi^{(1)} = \phi_0 \operatorname{sech}^2\left(\frac{\xi - u_0 \tau}{L_0}\right), \quad (46)$$

where $\phi_0 = 3u_0/A$, $L_0 = \sqrt{4B/u_0}$ and u_0 are the initial amplitude, width and velocity of the solitary wave in the absence of collisions, respectively.

For the KdV equation (45), the quantity

$$I = \int_{-\infty}^{+\infty} \left[\phi^{(1)}\right]^2 d\xi, \quad (47)$$

is conserved quantity.

Now, due to the presence of damping term $C\phi^{(1)}$, the solution of Eq. (44) is assumed to be exists in the form of solitary wave structures whose amplitude, width and velocity have a slowly dependency on time τ. Thus, the solitary wave solution of Eq. (44) can be approximated as [25]

$$\phi^{(1)} = \phi_m(\tau) \operatorname{sech}^2\left(\frac{\xi - u(\tau)\tau}{L(\tau)}\right), \quad (48)$$

where $\phi_m(\tau) = 3u(\tau)/A$, and $L(\tau) = \sqrt{4B/u(\tau)}$ represent the amplitude and width of the dissipative solitary wave as functions of time τ, respectively, and $u(\tau)$ its velocity. Now, differentiating (47) with respect to τ and using the Eq. (44) we fund the momentum conservation law as

$$\frac{dI}{d\tau} + 2CI = 0. \quad (49)$$

Substituting Eq. (48) into (47) we get $I = 24\sqrt{B}u(\tau)^{3/2}/A^2$. Using this into Eq. (49) and solving, we obtain an expression for $u(\tau)$ as

$$u(\tau) = u_0 \exp\left(-\frac{4C}{3}\tau\right). \quad (50)$$

Therefore, the time dependent expressions for amplitude and width of the dissipative solitary wave are given by $\phi_m = \phi_0 \exp\left(-\frac{4C}{3}\tau\right)$ and $L = L_0 \exp\left(\frac{2C}{3}\tau\right)$, respectively. Since the damping parameter C is positive [see Eq. (40)], damping term in Eq. (44) leads to a solitary wave collapsing with time.

5. Results and Discussion

The nonlinear properties of quantum DIAWs in a dense quantum DP system consisting of negatively charged dust grains, positive ions and dense quantum electrons and positrons are investigated, including the effects of electron/positron exchange-correlation potential and dust–neutral collisions. Using 2D cylindrical quantum hydrodynamic model, the nonlinear DCKP equation is derived using RPT. According to the dense dusty plasma characteristics, the physical quantities can be selected as:[26, 30] $n_{e0} = 0.5 \times 10^{30}\ m^{-3}$, $n_{i0} = 1.5 \times 10^{30}\ m^{-3}$, $Z_d = 10^4$, $n_{d0} = 1.4 \times 10^{25} m^{-3}$, $n_{p0} = 0.4 \times 10^{30} m^{-3}$. Figure 1 shows the dependent of phase velocity (w_0) quantum DIAWs on the equilibrium positron density [via the parameter $p(= n_{p0}/n_{e0})$] with fixed equilibrium electron density $n_{e0} = 0.5 \times 10^{30} m^{-3}$. Dashed curve is plotted with exchange-correlation effects, while the solid curve without exchange-correlation effects.

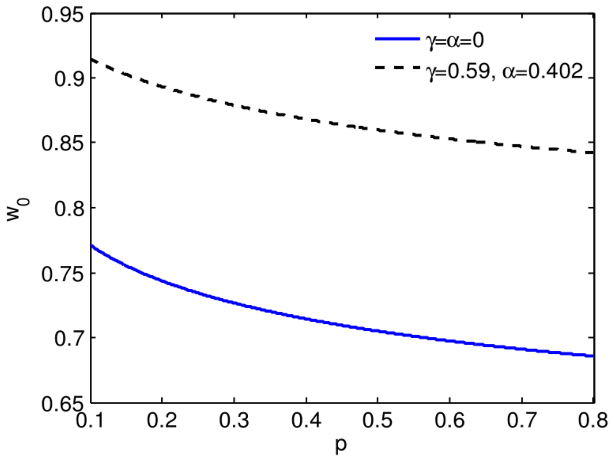

Figure 1: The variation of phase velocity w_0 against positrons density p, without ($\gamma = \alpha = 0$) and with ($\gamma = 0.59$, $\alpha = 0.4$) exchange-correlation potential, along with $n_{e0} = 0.5 \times 10^{30} m^{-3}$, $\mu_d = 0.1$, and $\mu = 0.6$.

From this figure one can see that the phase velocity w_0 decreases with increasing p in the presence or absence of exchange-correlation effects. The presence of exchange-correlation in the system leads to a wave with a greater phase velocity (see dashed curve).

Figures 2 and 3 illustrate the changes in the amplitudes ϕ_m and widths L of quantum dust ion acoustic (DIA) solitary waves due to the difference of p and $\mu(= n_{e0}/n_{i0})$ with fixed $n_{e0} = 0.5 \times 10^{30}\ m^{-3}$, respectively. Figure 2 indicates that the solitary wave amplitude ϕ_m decreases with both p and μ. For given values of the parameter μ, the amplitude ϕ_m decreases steeply with p for $p < 0.3$ and changes smoothly with p for $0.3 < p \leq 0.8$. For given values of p, the amplitude ϕ_m decreases with μ, which means that the DIA solitary wave amplitude ϕ_m increases with the larger values of the equilibrium ion density n_{i0} (via the parameter μ). On the other side, Fig. 3 indicates that for small values of $p < 0.5$, the solitary wave width L increases with p, and for greater values of $p \geq 0.5$, L decreases very smoothly with p. It is clear from Fig. 3 that, for given values of the parameter p, the width L decreases with μ. This means that, for larger values of ion density n_{i0}, the solitary wave width becomes larger.

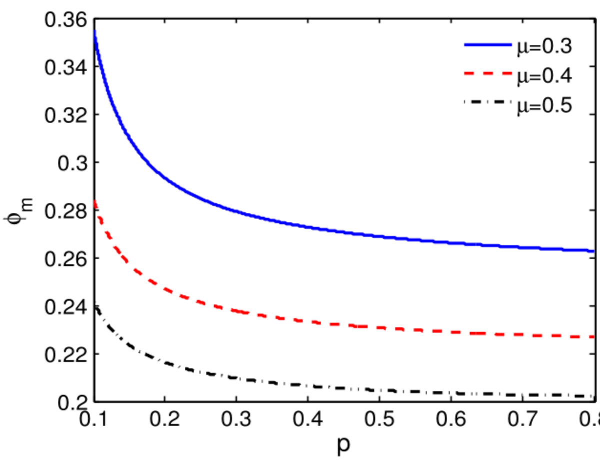

Figure 2: The variation of solitary wave amplitude $\phi_m(\tau)$ against positrons concentration p for different values of μ, along with $n_{e0} = 0.5 \times 10^{30} m^{-3}$, $\tau = 1$, $\mu_d = 0.1$, $H = 0.9$, $\nu_0 = 0.1$, $\alpha = 0.402$, $\gamma = 0.59$ and $u_0 = 0.1$

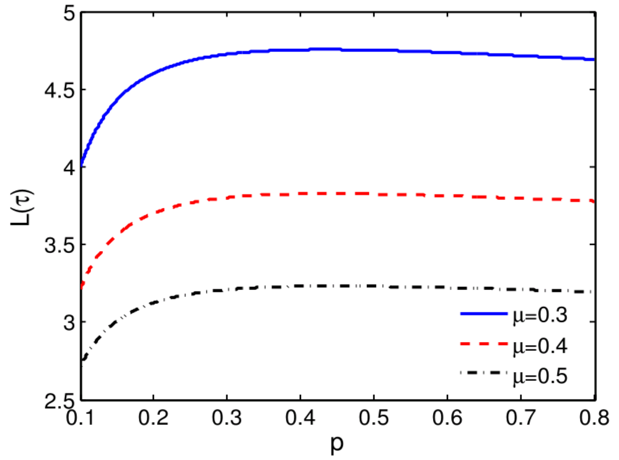

Figure 3. The variation of solitary wave width $L(\tau)$ against positrons concentration p for different values of μ. Other parameters same in Fig. 2.

In order to examine the effects of quantum diffraction and exchange-correlation on the dispersion properties of quantum DIA solitary waves, we plotted the width of the solitary wave versus quantum parameter H, for two cases, namely, in the presence and absence of an exchange-correlation effect as shown in the Fig. 4. It is clear from this figure that, in the both cases, the width of solitary wave is reduced by increasing the quantum parameter H. The presence of the exchange-correlation effects (via the parameters $\gamma = 0.59$ and $\alpha = 0.402$) leads to a solitary wave accompanied by a broader width.

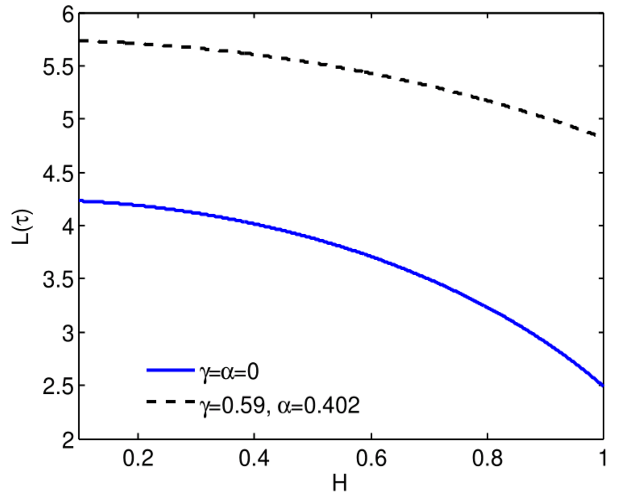

Figure 4: The variation of solitary width $L(\tau)$ against quantum parameter H, with and without exchange-correlation effect, along with $p = 0.6$, $\mu = 0.3$, $n_{e0} = 0.5 \times 10^{30}\ m^{-3}$, $\tau = 1$, $\mu_d = 0.1$.

Furthermore, Figure 5 indicates that the inclusion of the exchange-correlation effects in the system not only increases the solitary wave width but also leads to an increase in its amplitude.

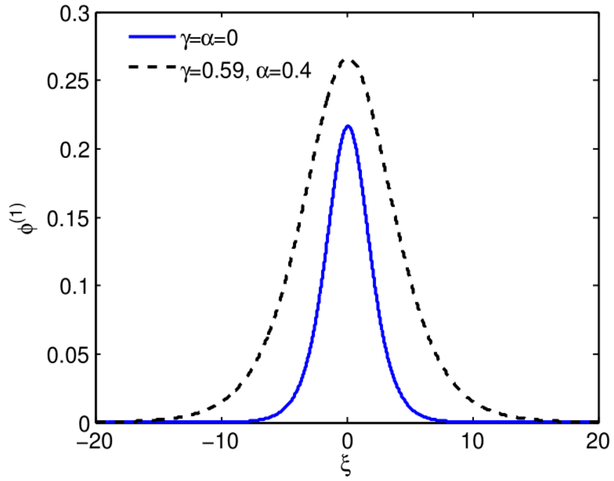

Figure 5: The effect of exchange-correlation on the solitary wave profile $\phi^{(1)}$, along with $p = 0.6$, $\mu = 0.3$, $\nu_0 = 0.1$, $\tau = 1$,$n_{e0} = 0.5 \times 10^{30}\ m^{-3}$, $\mu_d = 0.1$ and $u_0 = 0.1$

Figure 6 shows the time change of quantum solitary wave in the absence [Fig.5(a)] and presence [Fig. 5(b)] of dust-neutral collisions (via parameter ν_0).

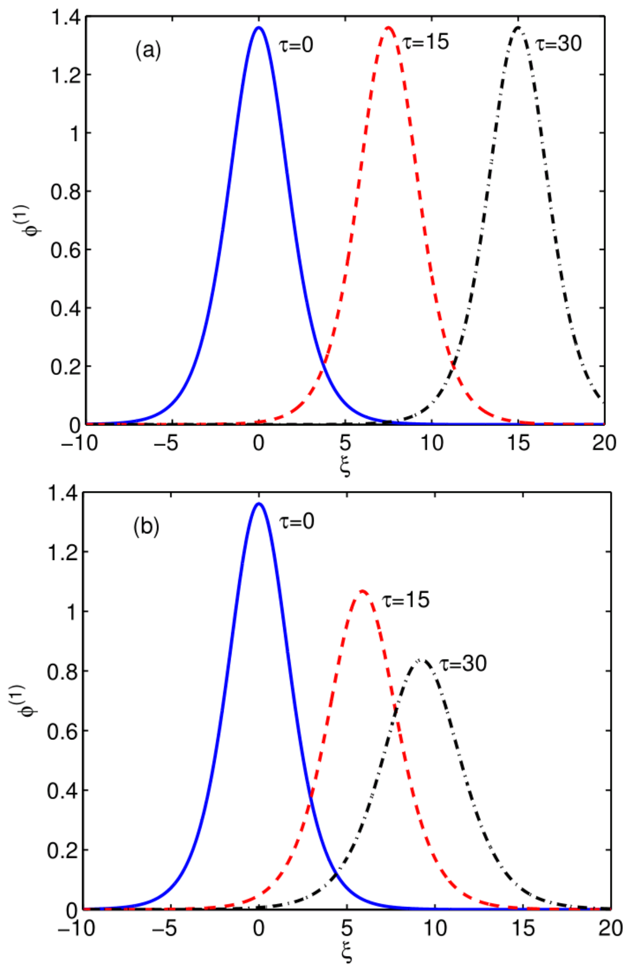

Figure 6: The profiles of the quantum DIASWs $\phi^{(1)}(\xi,\tau)$ against ξ, for different time. (a) is plotted in the absence of the collision ($\nu_0 = 0$) and (b) is plotted in the presence ($\nu_0 = 0.3$) of collision, along with $\alpha = 0.402$, $\gamma = 0.59$ and $u_0 = 0.5$. Other parameters same in Fig. 5.

In the absence of collisions (via $\nu_0 = 0$), Fig. 5(a) shows that as the time goes on (by increasing τ), the solitary wave travels forward in the ξ-direction without changing in its amplitude, width and velocity. On the other hand, in the presence of collision (via $\nu_0 = 0.3$), Fig. 5(b) indicates that the solitary wave amplitude decreases with time τ, accompanied by a reduced speed and an enlarged width. It is clear that the reason for the dissipative nature of the solitary wave is that its amplitude, width and velocity are all dependent on time. Such a solitary wave can travel a finite distance before collapsing at $\tau \to \infty$.

Now, to discuss the cylindrical geometry effect on the quantum DIA solitary waves, transforming the coordinates (ρ, ϑ, τ) back to the cylindrical coordinates (r, θ, t). Using Eqs. (41) and (42), the approximated solution (48) takes the form

$$\phi^{(1)}(\rho, \vartheta, \tau) = \phi_m(\tau)\,\mathrm{sech}^2\left\{\frac{1}{L(\tau)}\left[\rho - \left(u(\tau) + \frac{1}{2}w_0\vartheta^2\right)\tau\right]\right\}, \qquad (51)$$

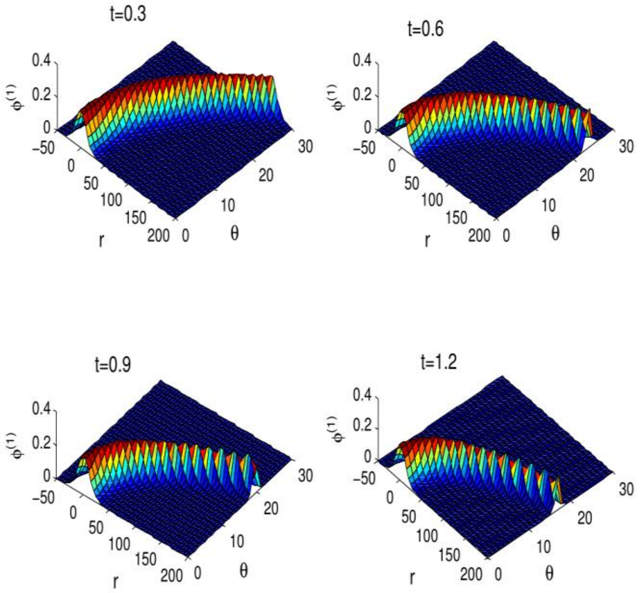

Figure 7: The time evolution of solitary wave pulses in the space r, θ with $p = 0.6$, $\mu = 0.3$. $n_{e0} = 0.5 \times 10^{30}\, m^{-3}$, $\alpha = 0.402$, $\gamma = 0.59$, $\mu_d = 0.1$, $\nu_0 = 0.1$, $\epsilon = 0.1$ and $u_0 = 0.1$.

where ρ, ϑ and τ are given by the stretch (18). Figure 7 displays the quantum DIA solitary wave as a function of radial (r) and angle (θ) coordinates for different times. It is noted that the propagation of solitary wave is shifted towards the radial direction with time going on. In fact, this behavior is due to the cylindrical geometry effects associated with transverse perturbation (via the θ-direction).

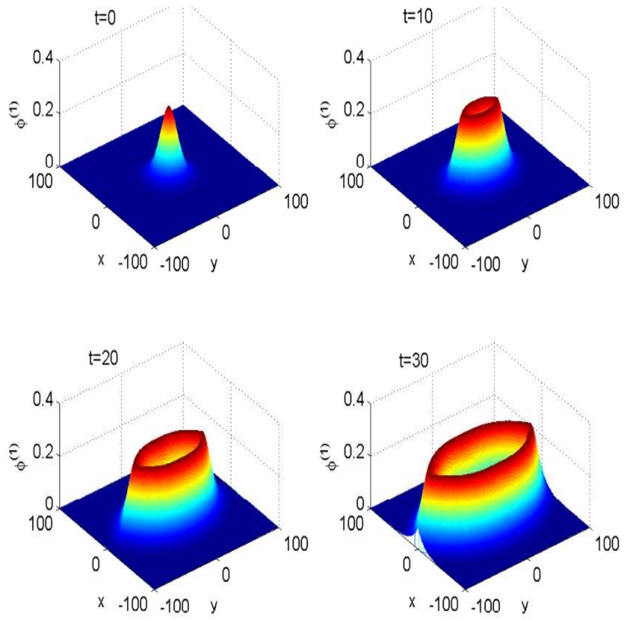

Figure 8: The time evolution of nebulon pulses corresponding to the Cartesian coordinates (x, y, t), along with $p = 0.6$, $\mu = 0.3$, $n_{e0} = 0.5 \times 10^{30}\, m^{-3}$, $\alpha = 0.402$, $\gamma = 0.59$, $\mu_d = 0.1$, $\nu_0 = 0.1$, $\epsilon = 0.1$ and $u_0 = 0.1$.

Because the Cartesian coordinates are the reference frame that generally considered in experiments, we have plotted the Figs. 8 and 9 using Eq. (51), by transforming the coordinates (ρ, ϑ, τ) to the cylindrical coordinates (r, θ, t), and then correspondingly to the Cartesian coordinates (x, y, t). Figure 8 exhibits the time evolution of nonlinear structures of quantum DIAWs in the space (x, y) for different times. It is noted from Fig. 8 that there is a new type of solitonic structures of quantum DIAWs may be formed (which is called nebulon) due to the Cartesian geometry and transverse perturbation. It is clear from this figure that the size of nebulon enlarges with time.

Figure 9 shows the contour plots of nebulon structures in the absence of exchange-correlation effects (i.e., when $\alpha = \gamma = 0$) Fig. 9(a), and in the presence of exchange-correlation effects (i.e., when $\alpha = 0.402$ and $\gamma = 0.59$) Fig. 9(b).

It can be seen from Fig. 9(b) that the nebulon structure becomes larger when the quantum exchange-correlation effects are considered in the system, but when the exchange-correlation potential effects are neglected (via $\alpha = \gamma = 0$), the nebulon structure becomes thinner as indicated in the Fig. 9(a).

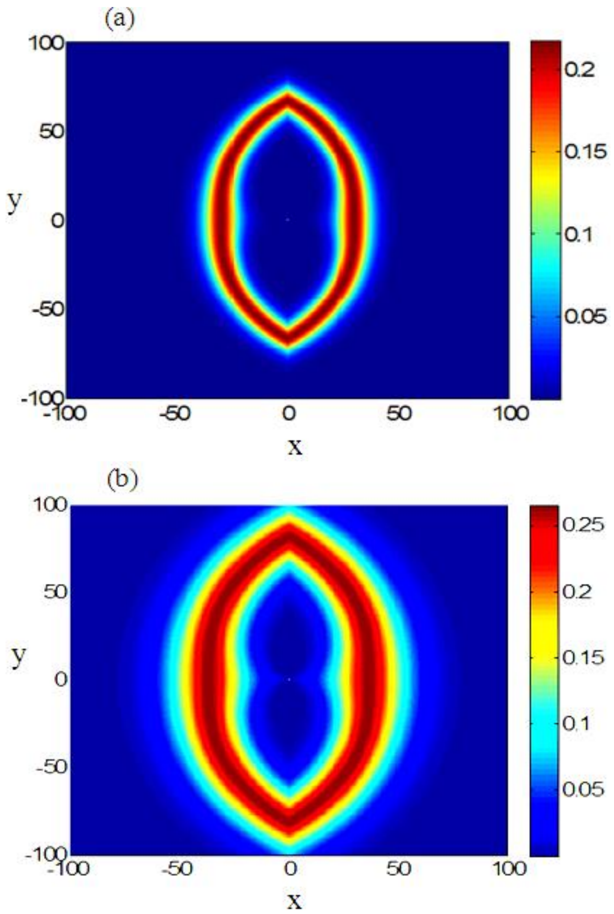

Figure 9: The contour plots of nebulon pulses. (a) without exchange-correlation effect (via $\alpha = \gamma = 0$) and (b) with exchange-correlation effect (via $\alpha = 0.402$ and $\gamma = 0.59$), along with $p = 0.6$, $\mu = 0.3$. $n_{e0} = 0.5 \times 10^{30}\ m^{-3}$, $\mu_d = 0.1$, $\nu_0 = 0.1$, $t = 30$, $\epsilon = 0.1$ and $u_0 = 0.1$.

6. Conclusions

Based on the 2D cylindrical geometry, the nonlinear properties of quantum DIAWs are studied in a quantum dusty plasma composed of a degenerate electrons and positrons, ions and negatively charged dust grains. Both negatively charged dust grains and ions are assumed to be classical and inertial while the electrons and positrons are assumed to be quantum and inertialess. Only dust-neutral collisions are considered which are found to provide dissipation. Using the reductive perturbation method, the DCKP equation is derived. The time dependent approximate solution of the DCKP equation is obtained as well. The effects of the quantum plasma parameters on the electrostatic quantum DIA solitary waves are examined. It is found that the quantum DIA solitary waves are significantly modified by the equilibrium positron concentration p, the quantum diffraction parameter H, and the exchange-correlation parameters. The geometrical effects on the profiles of quantum DIAWs have been investigated as well. It is found that the propagation of the solitary waves in a cylindrical geometry with weak transverse perturbations differs from that of a quasi-one-dimensional solitary wave. Further, it was found that the nebulon structures can exist associated with cylindrical quantum DIAWs and the main factors in their formation are Cartesian geometry and transverse perturbation. In addition, it was shown that, the quantum exchange-correlation parameters have noticeable effects on nebulon structures. Finally, the present investigations may be helpful in understanding the basic features of quantum DIAWs in dense astrophysical objects.

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