Vol.3, No.6, Page 1398- 1407, 2025 https://doi.org/10.59628/jast.v3i6.1940 ISSN 2958-9568 JAST

SUN PUBLISHER

مجلة جامعة صنعاء للعلوم التطبيقية والتكنولوجيا

Sana'a University Journal of Applied Sciences and Technology

https://journals.su.edu.ye/index.php/jast/

# Transverse Instability of Relativistic Ion-Acoustic Solitons in a Magnetized Space Plasmas with Heavy Ions and Superthermal Pair Electrons

**Hilal H. AL-Jawhari and Mahmood A. H. Khaled** *

**Department of Physics, Faculty of Sciences, Sana'a University, Sana'a, Yemen**

*Corresponding author: mahkhaled@su.edu.ye , mahkhaled@hotmial.com

## Abstract

A two-dimensional Zakharov Kuznetsov (ZK) equation was derived for ion-acoustic (IA) solitons in magnetoplasmas with relativistic streaming warm ions, superthermal electrons and positrons, and stationary heavy ions (either positive or negative). The transverse instability of the IA solitons was investigated using the small- $\kappa$ perturbation expansion method. The effects of plasma parameters such as the concentration of positive/negative heavy ions, relativistic streaming velocity of ions, superthermality of electrons and positrons, positron concentration, ion temperature, and magnetic field strength on the instability of IA solitons are investigated. These parameters have a significant effect on the instability growth rate. The results also indicate that the solitons tended to be stable in the plasma model containing negatively charged heavy ions. The findings of this work may provide valuable insights into the understanding of astrophysical systems, especially in the context of pulsar magnetospheres, solar wind, and Van Allen radiation belts, where relativistic ions and superthermal electron-positron coexist with heavy ions.

## Article Info

**Keywords:**
Zakharov-Kuznetsov (ZK) equation, heavy ions, ion-acoustic solitons, relativistic streaming ions.

**Article History:**
**Received:** 16-June-2025,
**Revised:** 11-July-2025,
**Accepted:** 15-August-2025,
**Available online:** 28 December 2025.

## 1. INTRODUCTION

Electron-positron-ion plasmas have unique properties that distinguish them from those of traditional electron-ion plasmas. Owing to the critical role that EPI plasmas play in understanding space and astrophysical environments, there has been considerable research interest in past years in characterizing nonlinear structures propagating in such plasmas [1–5]. Among these nonlinear structures are ion-acoustic (IA) solitons, which have been investigated by many authors in various EPI plasmas over the past years [6–11]. They showed that the dynamic characteristics of IA solitons are drastically altered by the presence of positrons in ordinary electron-ion plasma systems.

However, most observations on various astrophysical environments, such as solar wind [12], Earth's plasma sheet [13], and planetary magnetosphere [14, 15], have confirmed the presence of plasmas with superthermal particles, which have a significant influence on plasma dynamics. Such particles are characterized by pronounced high-energy tails in the velocity space, indicating a deviation from thermal equilibrium [16, 17]. In general, the most effective distribution for describing such energetic particles is the generalized Lorentzian (or kappa) distribution [18]. Several authors [17, 19–23] have shown that the presence of superthermal particles in EPI plasma environments has a significant impact on wave dynamics in such plasmas. They used kappa distributions to describe these superthermal particles and investigated the ion-acoustic waves (IAWs) in the framework of various nonlinear wave equations such as Korteweg-de Vries (KdV) [19], Kadomtsev-Petviashvili (KP) [20], and Zakharov-Kuznetsov (ZK) equations [21] or forced ZK equation [22]. Recently, El-Monier et al. [23] investigated nonlinear IAWs in dissipative magnetized EPI plasma with superthermal electrons and positrons.

On the other hand, heavy ion populations (i.e., ions with a mass significantly larger than that of protons, such as oxygen, carbon, iron, nitrogen, and other heavy metal ions) are common components of several regions of astrophysical environments, particularly in planetary magnetospheres, solar wind, and interstellar media [24–26]. The presence of heavy-ion populations in EPI plasma may have unique effects on the dynamics of the plasma system and on the properties of the nonlinear wave modes, including IA solitons. Hossen et al. [27] investigated the basic features of IA solitary and shock waves in a relativistic degenerate EPI plasma in the presence of positively charged static heavy ions. They found that the IA solitary and shock waves are significantly affected by the parameter of stationary heavy ions.

Numerous observations of space and astrophysical environments -including Earth's magnetospheric plasma sheet boundary layer [28], Van Allen radiation belts [29], and pulsar magnetospheres [30] have confirmed the existence of ions with energies ranging from to. Such ions can achieve velocities approaching the speed of light in vacuum. In this case, the relativistic effects are important and must be considered. Many investigations have been carried out in recent years to investigate the propagation characteristics of IA solitons in many relativistic EPI plasma models [31–38]. Some of these investigations were based on weakly relativistic effects [31–34], while others considered the influence of high relativistic [35–38]. However, all of these studies were accomplished using the KdV equation, which has stable plane soliton solutions. On the other hand, the nonlinear ZK equation is a very attractive model equation that can be used to describe the dynamical behavior of solitary waves or solitons in two or three dimensions. A few authors have investigated the basic properties of IA solitons in relativistic EPI plasmas by deriving a nonlinear ZK equation [39–41]. However, all of these investigations [39–41] were conducted outside the framework of the instability analyses of the soliton solution of ZK equation.

In general, the study of the transverse instability of the plane soliton solution of the ZK equation is very important because only stable solitons are promising for experimental observations and physical applications. Several investigations have been conducted on the instability analyses of soliton solutions of the ZK equation in different plasma models [42–51]. For example, the transverse instability of dust-acoustic solitary waves in a dusty plasma was analyzed through the derivation of the ZK equation that was solved numerically [48]. Through the derivation of ZK equation, Zedan et al. [49] investigated the multi-dimensional instability of IA solitons in a magnetized, degenerate multi-ion plasma using small-wavenumber (small- $\kappa$ ) perturbation expansion method. The plasma system consisted of inertial positive and negative ions, immobile negatively charged heavy ions, and trapping distributed electrons. Moreover, the instability and its growth rate of IAWs in a dense, unmagnetized multi-component plasma containing inertial negative and positive ions, as well as inertialess degenerate electrons and positrons were investigated by Khaled et al. [50]. More recently, Misra and Abdikian [51] studied the transverse instability of electron-acoustic solitons in a relativistic degenerate plasma using the small-perturbation expansion method.

To the best of our knowledge, no prior study has addressed the transverse instability of the planar soliton solution to the two-dimensional (2D) Zakharov-Kuznetsov (ZK) equation within a collisionless, magnetized multi-component plasma composed of relativistic warm light ions (hydrogen nuclei $H^+$ ), superthermally distributed electrons and positrons, and stationary heavy ions (either positively or negatively charged). Consequently, the primary objective of this paper is to investigate the transverse instability of IA solitons in this specific plasma configuration. To achieve this, we employ the reductive perturbation technique (RPT) [52], deriving the ZK equation which admits a solitary wave (or soliton) solution. Subsequently, we examine the transverse instability of this soliton solution by deriving the instability growth rate, adopting small-perturbation expansion method [42, 43]. Such plasmas may exist in space and astrophysical environments, especially the magnetosphere of pulsars, the solar wind, the Van Allen radiation belts, and planetary magnetospheres, where relativistic streaming ions and superthermal electrons and positron pairs can coexist with heavy ions.

The remainder of this paper is organized as follows: The governing equations for our plasma model are presented in Section 2. Using RPT, the 2D ZK equation is derived in Section 3, and the solution is determined in this section. Using the small-k perturbation method, the transverse instability and growth rate of IA solitons are investigated in Section 4. The results and discussion are presented in Section 5. The conclusions are presented in Section 6.

## 2. GOVERNING EQUATIONS

We consider a multi-component, magnetized, collisionless plasma model consisting of relativistic streaming warm positive light ions, superthermally distributed electrons and positrons, and immobile heavy ions (either positive or negative). We assume that such a plasma model is embedded in a uniform magnetic field oriented along the z-axis i.e., $\mathbf{B} = B_0\hat{e}_z$. Because the mass difference between heavy and light ions or electrons, the mobility of light ions may be much more than that of heavy ions. Therefore, we will neglect the mobility of heavy ions and consider them as a stationary background. Furthermore, we consider that the relativistic streaming ion velocity is in the z-direction only. For such plasmas, the quasi-neutrality condition is: $n_{e0} + JZ_h n_{h0} - Z_i n_{i0} - n_{p0} = 0$, where $n_{e0}$, $n_{p0}$, $n_{i0}$ and $n_{h0}$ represent the unperturbed densities of the electrons, positrons, light ions, and stationary heavy ions, respectively. $Z_h$ and $Z_i$ are the electronic charges of the heavy and light ions, respectively, and $J = -1\,(+1)$ for positive (negative) heavy ions. For relativistic streaming of light ions, we deal with hydrogen nuclei $H^+$ (primarily protons). So, we take $Z_i = +1$. Accordingly, the propagation of IAWs in such plasmas is described by the following normalized equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i u_{ix}\right) + \frac{\partial}{\partial z}\left(n_i u_{iz}\right) = 0, \quad (1)$$

$$\frac{\partial u_{ix}}{\partial t} + u_{ix}\frac{\partial u_{ix}}{\partial x} + u_{iz}\frac{\partial u_{ix}}{\partial z} = -\frac{\partial \phi}{\partial x} - \frac{\sigma}{n_i}\frac{\partial p_i}{\partial x} + u_{iy}\Omega_c, \quad (2)$$

$$\frac{\partial u_{iy}}{\partial t} + u_{ix}\frac{\partial u_{iy}}{\partial x} + u_{iz}\frac{\partial u_{iy}}{\partial z} = -u_{ix}\Omega_c, \quad (3)$$

$$\frac{\partial \gamma u_{iz}}{\partial t} + u_{ix}\frac{\partial \gamma u_{iz}}{\partial x} + u_{iz}\frac{\partial \gamma u_{iz}}{\partial z} = -\frac{\partial \phi}{\partial z} - \frac{\sigma}{n_i}\frac{\partial p_i}{\partial z}, \quad (4)$$

$$\frac{\partial p_i}{\partial t} + u_{ix}\frac{\partial p_i}{\partial x} + u_{iz}\frac{\partial p_i}{\partial z} + 2p_i\left(\frac{\partial u_{ix}}{\partial x} + \frac{\partial \gamma u_{iz}}{\partial z}\right) = 0, \quad (5)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + n_i = \alpha_e n_e - \alpha_p n_p + j\alpha_h, \quad (6)$$

where $n_i$ and $p_i$ are the number density and thermal pressure of positive light ions, normalized by $n_{i0}$, $n_{i0}k_B T_i$, respectively. The quantity $\phi$ represents the electrostatic potential, normalized by $m_i c_s^2/e$, where $c_s = \sqrt{k_B T_e/m_i}$ is the ion sound speed, $m_i$ is the ion mass, $T_i(T_e)$ is the ion (electron) temperature, and $k_B$ is the Boltzmann constant. The quantities $u_{ix}$, $u_{iy}$, and $u_{iz}$ represent the light- ion fluid velocities in the $x$, $y$, and $z$ directions, respectively, and are normalized by the ion sound speed $c_s$. The space $(x, z)$ and time $(t)$ variables are normalized by Debye length $\lambda_D = \sqrt{\epsilon_0 k_B T_e/n_{i0}e^2}$ and inverse ion plasma frequency $\omega_{pi}^{-1} = \left(n_{i0}e^2/\epsilon_0 m_i\right)^{-1/2}$, respectively, where $\epsilon_0$ is the permittivity of vacuum. The quantity $\Omega_c$ represents the dimensionless ion gyrofrequency, defined as $\Omega_c = \omega_{ci}/\omega_{pi}$, where $\omega_{ci} = eB_0/m_i$ is the dimensional ion gyrofrequency. We define $\sigma = T_i/T_e$ as the ion- to- electron temperature ratio, $\sigma_p = T_e/T_p$ as the electron-to-positron temperature ratio, $\alpha_e = n_{e0}/n_{i0} = (1 - J\alpha_h)/(1 - \delta)$, and $\alpha_p = n_{p0}/n_{i0} = \delta\,(1 - J\alpha_h)/(1 - \delta)$ where $\delta = n_{p0}/n_{e0}$ is the positron-to-electron density ratio, and $\alpha_h = n_{h0}/n_{i0}$ is the heavy-to-light ion density ratio. The relativity factor $\gamma = \left(1 - u_{iz}^2/c^2\right)^{-1/2}$ appearing in Eqs. Eq. (4) and Eq. (5) is approximated as in $\gamma \approx 1 + u_{iz}^2/2c^2 + 3u_{iz}^4/8c^4$.

Because the electrons and positrons in our magnetoplasma model are assumed to follow kappa distributions, their densities are given by the following normalized expressions: [21–23]

$$n_e = \left(1 - \frac{\phi}{\kappa_e - 3/2}\right)^{-\kappa_e + \frac{1}{2}} \quad (7)$$

$$n_p = \left(1 + \frac{\sigma_p \phi}{\kappa_p - 3/2}\right)^{-\kappa_p + \frac{1}{2}} \quad (8)$$

where $\kappa_e$ and $\kappa_p$ are the superthermal parameters of the electrons and positrons, respectively. It should be noted that at very large values of $\kappa_j$–parameter (i.e., when $\kappa_j \to \infty$), the kappa distribution is close to the Maxwellian distribution (i.e., close to the thermal equilibrium state), whereas for smaller values of $\kappa_j$ , they present a hard spectrum with a strong non-Maxwellian tail having a power law form at high speed.

Because the electrostatic potential perturbation is very small (i.e., $\phi \ll 1$), the densities of electrons and positrons in Eqs. Eq. (7) and Eq. (8) are expanded using a Taylor expansion around $\phi = 0$ to $\phi^2$. With these expansions, Poisson's equation Eq. (6) can be simplified to:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + n_i = 1 + a_1\phi + a_2\phi^2 + \ldots \quad (9)$$

where the coefficients $a_1$, and $a_2$ are calculated to be

$$a_1 = \left(\frac{1 - J\alpha_h}{1 - \delta}\right)\left[\frac{(\kappa_e - 1/2)}{\kappa_e - 3/2} + \frac{\delta\sigma_p\left(\kappa_p - 1/2\right)}{\kappa_p - 3/2}\right] \quad (10)$$

and

$$a_2 = \frac{1}{2}\left(\frac{1 - J\alpha_h}{1 - \delta}\right)\left[\frac{\left(\kappa_e^2 - 1/4\right)}{\left(\kappa_e - 3/2\right)^2} - \frac{\delta\sigma_p^2\left(\kappa_p^2 - 1/4\right)}{\left(\kappa_p - 3/2\right)^2}\right]. \quad (11)$$

## 3. DERIVATION OF ZK EQUATION

To derive the 2D ZK equation, which evinces the propagation of small-amplitude IA solitons, RPT is used [52]. Accordingly, we consider the following transformations.

$$\xi = \epsilon^{1/2}x, \;\; \zeta = \epsilon^{1/2}\left(z - V_p t\right), \quad (12)$$

$$\tau = \epsilon^{3/2}t, \quad (13)$$

where $V_p$ is the linear phase velocity of IAW (to be determined) and $\epsilon$ is small real parameter i.e., $0 < \epsilon < 1$. The normalized quantities $n_i$, $u_{iz}$, $p_i$ and $\phi$ are expanded in terms of power series in $\epsilon$ as

$$\begin{pmatrix} n_i \\ u_{iz} \\ p_i \\ \phi \end{pmatrix} = \begin{pmatrix} 1 \\ u_0 \\ 1 \\ 0 \end{pmatrix} + \epsilon\begin{pmatrix} n_1 \\ u_{z1} \\ p_1 \\ \phi_1 \end{pmatrix} + \epsilon^2\begin{pmatrix} n_2 \\ u_{z2} \\ p_2 \\ \phi_2 \end{pmatrix} + \ldots, \quad (14)$$

while the quantities $u_{ix}$ and $u_{iy}$ are expanded as follows

$$u_{ix} = \epsilon^2 u_{x1} + \epsilon^3 u_{x2} + \ldots, \quad (15)$$

$$u_{iy} = \epsilon^{3/2}u_{y1} + \epsilon^{5/2}u_{y2} + \ldots. \quad (16)$$

Substituting the expressions Eq. (12)-Eq. (16) into the normalized basic equations Eq. (1) to Eq. (5) and Eq. (9), and then collecting the terms for different powers of $\epsilon$, and setting them equal to zero, we obtain a set of equations in different orders in $\epsilon$. For lower- order equations, that is , $O(\epsilon^{3/2})$, we obtain the following first-order perturbative quantities:

$$n_1 = \frac{1}{\gamma_1\left(\lambda^2-2\sigma\right)}\phi_1, \tag{17}$$

$$u_{z1} = \frac{\lambda}{\gamma_1\left(\lambda^2-2\sigma\right)}\phi_1, \tag{18}$$

$$p_1 = \frac{2}{\lambda^2-2\sigma}\phi_1, \tag{19}$$

$$n_1 = a_1\phi_1, \tag{20}$$

where $\lambda = V_p - u_0$ is the relativistic linear phase velocity of IAWs, and $\gamma_1 = 1+3\beta_0^2/2+15\beta_0^4/8$ is a parameter related to the relativistic effect, and $\beta_0 = u_0/c$ is a relativistic parameter.
Substituting Eq. Eq. (17) into Eq. Eq. (20), we obtain the relativistic linear phase velocity of IAWs as

$$\lambda = \sqrt{\frac{1}{\gamma_1 a_1}+2\sigma}. \tag{21}$$

Moreover, the lowest order terms in $\epsilon$ for $x$ and $y$ components of momentum equations lead to the following relations:

$$u_{x1} = \frac{\lambda}{\Omega_c}\frac{\partial u_{y1}}{\partial \zeta}, \tag{22}$$

$$u_{y1} = \frac{\lambda^2}{\Omega_c\left(\lambda^2-2\sigma\right)}\frac{\partial \phi_1}{\partial \xi}. \tag{23}$$

Equations Eq. (22) and Eq. (23) give

$$u_{x1} = \frac{\lambda^3}{\Omega_c^2\left(\lambda^2-2\sigma\right)}\frac{\partial^2 \phi_1}{\partial\zeta\partial\xi}. \tag{24}$$

For the second-order terms $(\epsilon^2)$ of the Poisson equation Eq. (9), we obtain the following equation:

$$n_2 = a_1\phi_2 + a_2\phi_1^2 - \frac{\partial^2\phi_1}{\partial\xi^2} - \frac{\partial^2\phi_1}{\partial\zeta^2}. \tag{25}$$

Now, by collecting the next higher-order terms of $\epsilon$ (i.e., $\epsilon^{5/2}$) for the continuity, z-component momentum, and energy equations, we obtain

$$\frac{\partial n_1}{\partial\tau} - \lambda\frac{\partial n_2}{\partial\zeta} + \frac{\partial u_{z2}}{\partial\zeta} + \frac{\partial u_{x1}}{\partial\xi} + \frac{\partial n_1 u_{z1}}{\partial\zeta} = 0, \tag{26}$$

$$\gamma_1\frac{\partial u_{z1}}{\partial\tau} - \gamma_1\lambda\frac{\partial u_{z2}}{\partial\zeta} - 2\gamma_2\lambda u_{z1}\frac{\partial u_{z1}}{\partial\zeta} + n_1\frac{\partial\phi_1}{\partial\zeta} + \frac{\partial\phi_2}{\partial\zeta} + \sigma\frac{\partial p_2}{\partial\zeta} = 0, \tag{27}$$

$$\frac{\partial p_1}{\partial\tau} - \lambda\frac{\partial p_2}{\partial\zeta} + u_{z1}\frac{\partial p_1}{\partial\zeta} + 2\gamma_1\frac{\partial u_{z2}}{\partial\zeta} + 4\gamma_2 u_{z1}\frac{\partial u_{z1}}{\partial\zeta} + 2\gamma_1 p_1\frac{\partial u_{z1}}{\partial\zeta} + 2\frac{\partial u_{x1}}{\partial\xi} = 0, \tag{28}$$

where parameter $\gamma_2$ is also related to relativistic effects and is defined by $\gamma_2 = 3\beta_0/2c + 15\beta_0^3/2c$. Substituting the first-order quantities $n_1$, $u_{z1}$, $p_1$ and $u_{x1}$ from Eqs. (17)–(20) and Eq. (24) into the Eqs. (26-28), we get

$$a_1\frac{\partial\phi_1}{\partial\tau} + 2a_1^2\lambda\phi_1\frac{\partial\phi_1}{\partial\zeta} + \frac{\gamma_1 a_1\lambda^3}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^3} - \lambda\frac{\partial n_2}{\partial\zeta} + \frac{\partial u_{z2}}{\partial\zeta} = 0, \tag{29}$$

$$\gamma_1 a_1\lambda\frac{\partial\phi_1}{\partial\tau} + a_1\left(1-2\gamma_2 a_1\lambda^3\right)\phi_1\frac{\partial\phi_1}{\partial\zeta} - \gamma_1\lambda\frac{\partial u_{z2}}{\partial\zeta} + \frac{\partial\phi_2}{\partial\zeta} + \sigma\frac{\partial p_2}{\partial\zeta} = 0, \tag{30}$$

$$2\gamma_1 a_1\frac{\partial\phi_1}{\partial\tau} + 2a_1^2\lambda\left[\gamma_1\left(1+2\gamma_1\right)+2\gamma_2\lambda\right]\phi_1\frac{\partial\phi_1}{\partial\zeta} + \frac{2\gamma_1 a_1\lambda^3}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^3} + 2\gamma_1\frac{\partial u_{z2}}{\partial\zeta} - \lambda\frac{\partial p_2}{\partial\zeta} = 0, \tag{31}$$

where we have used $\gamma_1\left(\lambda^2-2\sigma\right) = 1/a_1$ from Eq. Eq. (21). Now, eliminating $p_2$ between Eqs. (30) and (31) gives us

$$\gamma_1 a_1\left(\lambda^2+2\sigma\right)\frac{\partial\phi_1}{\partial\tau} + a_1\lambda\left[1+2\sigma a_1\gamma_1\left(1+2\gamma_1\right)-\frac{2\gamma_2\lambda}{\gamma_1}\right]\phi_1\frac{\partial\phi_1}{\partial\zeta} + \frac{2\sigma\gamma_1 a_1\lambda^3}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^3} - \frac{1}{a_1}\frac{\partial u_{z2}}{\partial\zeta} + \lambda\frac{\partial\phi_2}{\partial\zeta} = 0, \tag{32}$$

and again eliminating $u_{z2}$ between Eqs. (29) and (32) gives the following equation:

$$2\gamma_1 a_1\lambda^2\frac{\partial\phi_1}{\partial\tau} + a_1\lambda\left[3+2\sigma a_1\gamma_1\left(1+2\gamma_1\right)-\frac{2\gamma_2\lambda}{\gamma_1}\right]\phi_1\frac{\partial\phi_1}{\partial\zeta} + \frac{\lambda^3\gamma_1\left(2\sigma a_1+1\right)}{\Omega_c^2}\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^2} - \frac{\lambda}{a_1}\frac{\partial n_2}{\partial\zeta} + \lambda\frac{\partial\phi_2}{\partial\zeta} = 0, \tag{33}$$

Finally, we substitute $n_2$ from Eq. Eq. (25) into Eq. Eq. (33), the following 2D ZK equation is obtained:

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\zeta} + B\frac{\partial^3\phi_1}{\partial\zeta^3} + D\frac{\partial^3\phi_1}{\partial\zeta\partial\xi^2} = 0, \tag{34}$$

which describes the propagation of IA solitons in a 2D relativistic magnetized plasma containing relativistic streaming light ions, stationary heavy ions, superthermal electrons, and positrons. Coefficient $A$ represents a nonlinear coefficient, whereas coefficients $B$ and $D$ represent the dispersion coefficients in the parallel and transverse directions of the external magnetic field, respectively, which are defined by

$$A = \frac{1}{2\lambda\gamma_1}\left[3+2\sigma a_1\gamma_1\left(1+2\gamma_1\right)-\frac{2\gamma_2\lambda}{\gamma_1}-2\frac{a_2}{a_1^2}\right],$$

$$B = \frac{1}{2\lambda\gamma_1 a_1^2},$$

$$D = B\left[1 + \frac{\lambda^2\gamma_1 a_1\left(2\sigma a_1 + 1\right)}{\Omega_c^2}\right].$$

To obtain the stationary plane soliton solution of the 2D ZK equation Eq. (34), the following transformations are introduced:

$$Z = \zeta - U\tau,\ \ X = \xi,\ \ T = \tau, \tag{35}$$

where $U$ is a constant speed normalized by $c_s$. With the transformations Eq. (35), the 2D ZK equation Eq. (34) becomes

$$\frac{\partial\phi_1}{\partial T} - U\frac{\partial\phi_1}{\partial Z} + A\phi_1\frac{\partial\phi_1}{\partial Z} + B\frac{\partial^3\phi_1}{\partial Z^3} + D\frac{\partial^2\phi_1}{\partial Z\partial X^2} = 0. \tag{36}$$

If we consider $\phi_1 = \phi_0\left(Z\right)$ to be a steady-state solution of the ZK equation, the above equation reduces to the KdV equation as

$$-U\frac{d\phi_0}{dZ} + A\phi_0\frac{d\phi_0}{dZ} + B\frac{d^3\phi_0}{dZ^3} = 0. \tag{37}$$

Integrating and applying the boundary conditions: $\phi_0,\ d\phi_0/dZ$ , $d^2\phi_0/dZ^2 \to 0$ as $Z \to \pm\infty$, we obtain the following one soliton solution:

$$\phi_0\left(\mathrm{Z}\right) = A_0\mathrm{sech}^2\left(Z/w\right), \tag{38}$$

where $A_0$ and $w$ are the amplitude and width of the IA solitons propagating along the $z$ axis, respectively, and are given as:

$$A_0 = 3U/A,\ \ w = 2(B/U)^{1/2}. \tag{39}$$

## 4. INSTABILITY ANALYSIS

To analyze the instability of the plane soliton solution of the 2D ZK equation, we employed the small-$k$ perturbation expansion method [42, 43]. Therefore, we consider the solution to Eq. (36), as follows:

$$\phi_1 = \phi_0 + \epsilon\psi\left(Z\right)e^{ikX}e^{\Lambda T}, \tag{40}$$

where $k$ represents the wavenumber in the transverse direction (i.e., $x$-axis), and $\Lambda$ represents the instability growth rate of IA solitons, which occurs when $\mathrm{Re}\left(\Lambda\right) \neq 0$. Substituting Eq. (40) into Eq. (36) and neglecting the second- order terms in $\epsilon$ ($\epsilon^2$), we obtain:

$$\frac{d}{dZ}\left(\widehat{\mathrm{H}}\psi\right) = Dk^2\frac{d\psi}{dZ} - \Lambda\psi, \tag{41}$$

where the operator $\widehat{\mathrm{H}}$ is defined by

$$\widehat{\mathrm{H}} = B\frac{d^2}{dZ^2} + A\phi_0 - U.$$

For long wavelengths, we can expand $\psi\left(Z\right)$ and $\Lambda$ can be expanded in terms of $k$ as follows:

$$\psi = \psi_0 + k\psi_1 + k^2\psi_2 + \ldots, \tag{42}$$

$$\Lambda = 0 + k\Lambda_1 + k^2\Lambda_2 + \ldots. \tag{43}$$

Substituting Eq. (42) and Eq. (43) into Eq. (41), and equating the ascending powers of $k$ in the resulting equation leads to a series of equations that are used to determine $\psi_0,\ \psi_1, \ldots$, and then $\Lambda_1,\ \Lambda_2,\ \ldots$. For the zero-order terms in $k$ , we obtain the following equation:

$$\frac{d}{dZ}\left(\widehat{\mathrm{H}}\psi_0\right) = 0. \tag{44}$$

To obtain a solution to Eq. (44), we return to Eq. (37), and write it in the following form:

$$\widehat{\mathrm{H}}\frac{d\phi_0}{dZ} = 0. \tag{45}$$

Differentiating the above equation, one concerning the variable $Z$ we get

$$\frac{d}{dZ}\left(\widehat{\mathrm{H}}\frac{d\phi_0}{dZ}\right) = 0. \tag{46}$$

Comparing Eq. (44) and Eq. (46), the solution to Eq. (44) becomes

$$\psi_0 = \frac{d\phi_0}{dZ}. \tag{47}$$

To determine the function $\psi_1\left(Z\right)$, we check the first-order terms in $k$, which yields

$$\frac{d}{dZ}\left(\widehat{\mathrm{H}}\psi_1\right) = -\Lambda_1\psi_0. \tag{48}$$

By substituting $\psi_0$ from Eq. Eq. (47) into Eq. (48), and then integrating once with the boundary conditions: the function $\psi_1$ and its derivatives vanish as $|Z| \to \infty$, we obtain the following differential equation:

$$\widehat{\mathrm{H}}\psi_1 = -\Lambda_1\psi_0,$$

which can be written in the form

$$\frac{d^2\psi_1}{dZ^2} + \left(A\phi_0 - U\right)\psi_1 = -\Lambda_1\psi_0. \tag{49}$$

Equation Eq. (49) is an inhomogeneous second-order differential equation: According to Eq. (37), the homogeneous solution to Eq. (49) becomes $d\phi_0/dZ$ . Therefore, it is appropriate to write the solution to Eq. (49) in the following form:

$$\psi_1 = G\left(Z\right)\frac{d\phi_0}{dZ}, \tag{50}$$

where function $G(Z)$ must be determined. By substituting the solution Eq. (50) into Eq. (49). After performing some mathematical calculations, we finally obtain the expression of the function $G(Z)$ as follows:

$$G\left(Z\right) = -\frac{w^2\Lambda_1}{8B}\left[Z - w\coth\left(\frac{Z}{w}\right)\right]. \tag{51}$$

Substituting into Eq. (50), the solution of Eq. (49) becomes

$$\psi_1 = \frac{\Lambda_1 w}{4B} A_0 \text{sech}^2\left(\frac{Z}{w}\right)\left[Z\tanh\left(\frac{Z}{w}\right) - w\right]. \quad (52)$$

Now, for the second order terms in $k$, we obtain

$$\frac{d}{dZ}\left(\widehat{\mathrm{H}}\psi_2\right) = D\frac{d\psi_0}{dZ} - \Lambda_2\psi_0 - \Lambda_1\psi_1. \quad (53)$$

For a solution of Eq. (53) to exist, the right-hand side must be orthogonal to kernel $\phi_0$, where this kernel vanishes as $Z \to \pm\infty$. In other words, the existence of a solution to Eq. (53) requires the following condition to be satisfied.

$$\int_{-\infty}^{+\infty} \phi_0 \left(D\frac{d\psi_0}{dZ} - \Lambda_2\psi_0 - \Lambda_1\psi_1\right) dZ = 0. \quad (54)$$

Substituting $\phi_0$, $\psi_0$, and $\psi_1$ from Eq. (38), Eq. (47), and Eq. (52), and then performing integration, we obtain

$$\Lambda_1 = \frac{2U}{\sqrt{15}}\sqrt{D/B}. \quad (55)$$

Equation (55) is the first-order instability growth rate of the soliton solution of the ZK equation in the transverse direction, which depends on the plasma parameters via dispersive coefficients $B$ and $D$. In particular, if we take $B = D = 1$, we obtain $\Lambda_1 = 2U/\sqrt{15}$. This result is quite similar to those obtained by Akhtar *et al.* [47].

## 5. NUMERICAL RESULTS AND DISCUSSION

In this work, we have considered an astrophysical magneto-plasma with relativistic streaming light ions ($H^+$), superthermality distributed electrons, and positrons, in addition to a stationary heavy ions background (positive or negative). Employing the RPT, a two-dimensional ZK equation was derived to investigate the transverse instability analysis of small- amplitude IA solitons. Using the small-$k$ perturbation expansion method, the first- order instability growth rate $\Lambda_1$ for the IA solitons was determined. Figures 1(a) and 1(b) depict the variation in the instability growth rate $\Lambda_1$with superthermality of electrons (via $\kappa_e$-index) and positrons (via $\kappa_p$-index), respectively, for different values of ambient magnetic field strength (via $\Omega_c$-parameter), in the presence of positively charged heavy ions in the model. It is clear from these figures that the instability growth rate $\Lambda_1$ decreases with increasing ambient magnetic field strength and superthermality of electrons and positrons. For a fixed value of $\Omega_c$, the growth rate $\Lambda_1$ is unaffected by higher values of $\kappa_e$ or $\kappa_p$-index. This result was expected because the plasma model may reach its thermal equilibrium state at higher superthermal index values ($\kappa_e$ and $\kappa_p$).

However, for lower values of $\kappa_e$ and $\kappa_p$-indexes, the growth rate $\Lambda_1$ decreases rapidly with $\kappa_e$ or $\kappa_p$-index. This is because the smaller values of $\kappa_{e,p}$ correspond to

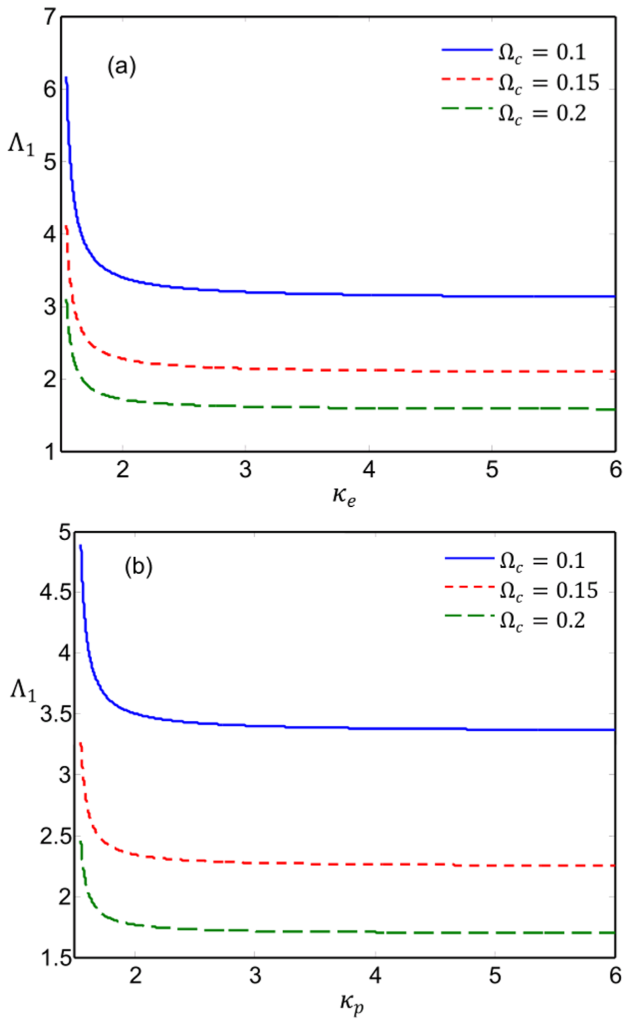


**Figure 1.** The variation of the growth rate $\Lambda_1$ versus the superthermal parameters $\kappa_e, \kappa_p$ for different values of $\Omega_c$ in the presence of positively charged heavy ions. (a) is plotted with superthermal parameters of electrons $\kappa_e$with $\kappa_p = 1.8$; and (b) with superthermal parameters of positions $\kappa_p$ with $\kappa_e = 1.8$. Other parameters are $\sigma_p = 1$, $\sigma = 0.01$, $\alpha_p = 0.3$, $\beta_0 = 0.4$, and $\delta = 0.5$.

the furthest state from thermal equilibrium, and the kappa (or superthermal) distribution is suitable for describing the behavior of plasma particles. Figure 2 illustrates the variation in the instability growth rate $\Lambda_1$ as a function of superthermal index $\kappa$ for electrons and positrons in the presence of positively charged heavy ions (dashed line), and negatively charged heavy ions (solid line). The figure shows that the growth rate $\Lambda_1$ decreases with increasing superthermal index $\kappa$, whether in the presence of positively or negatively charged heavy ions in the plasma model. However, in the plasma model characterized by the presence of positively charged heavy ions, an enhancement in the growth rate is evident, in contrast to the model involving negatively charged heavy ions, where $\Lambda_1$ decreas significantly. This behavior may be attributed to the fact that the presence of positively charged heavy ions in the model may contribute more effectively to the nonlinear dynamics of the plasma model, thereby supporting the instability of the solitons. Conversely, the presence of negatively charged heavy ions tends to stabilize the system, promoting stable solitons.

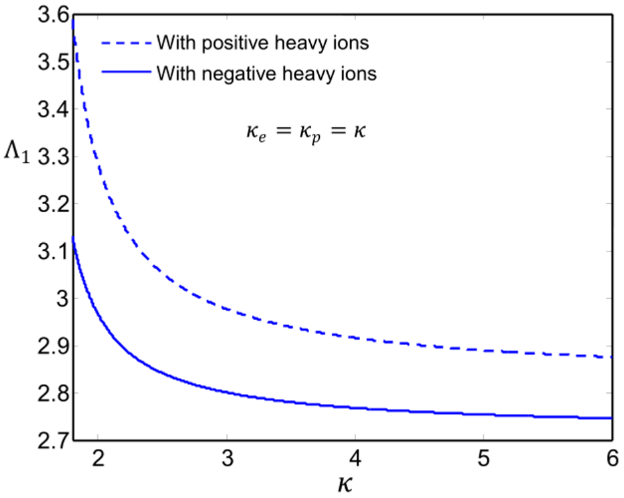


**Figure 2.** The variation of the growth rate $\Lambda_1$ versus the $\kappa$-index of superthermal electrons and positrons, for two plasma models: One with positive heavy ions (dashed curve) and the other with negative heavy ions (solid curve), along with $\sigma_p = 1$, $\sigma = 0.01$, $\alpha_h = 0.3$, $\beta_0 = 0.4$, $U = 0.5$, $\Omega_c = 0.1$, and $\delta = 0.5$.

Figures 3(a) and 3(b) show the variation in the instability growth rate ( $\Lambda_1$ ) as a function of the relativistic streaming parameter ($\beta_0$) for different values of the concentration of negative (Fig.3a), and positively (Fig. 3b) Charged heavy ions. It is found from the figures that an increase in the $\beta_0$-parameter leads to an increase in the instability growth rate $\Lambda_1$. Figure 3(a) demonstrates that incorporating negative heavy ions into the plasma model significantly alters the instability growth rate. Specifically, the growth rate exhibits a gradual reduction with increasing concentrations of negative heavy ions $\alpha_h$. In contrast, Fig. 3(b) reveals that the inclusion of positively charged heavy ions results in a marked increase in the instability growth rate$\Lambda_1$. The variation in the instability growth rate $\Lambda_1$ as a function of the relativistic parameter $\beta_0$ is shown in Fig. 4 for both superthermal and Maxwellian distributions of electrons and positrons, in the presence of positively/negatively charged heavy ions. It can be seen from the figure that the instability growth rate $\Lambda_1$ increases with $\beta_0$-parameter for both electron-positron distributions. However, at the Maxwellian limit (where $\kappa_e$and $\kappa_p \rightarrow \infty$), the effect of $\beta_0$ becomes significantly weaker. It was also observed that the growth rate $\Lambda_1$ increased (decreased) in the presence of positive (negative) heavy ions.

Figure 5 depicts how the instability growth rate $\Lambda_1$ is affected by increasing the positrons concentration (via $\delta$-parameter) for the two plasma models (**one contain-**

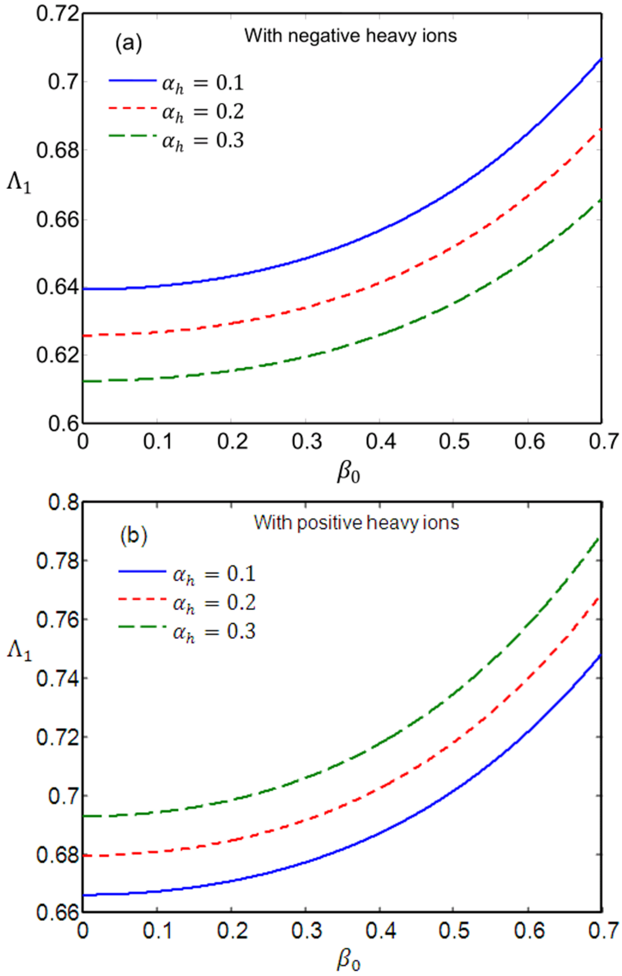


**Figure 3.** The variation of the growth rate $\Lambda_1$ versus the relativistic parameter $\beta_0$, for different values of the concentration of negative (a) and positive (b) heavy ions $\alpha_h$, along with $\sigma_p = 1$, $\sigma = 0.01$, $\alpha_p = 0.5$, $\kappa_e = \kappa_p = 1.8$, $U = 0.1$, and $\Omega_c = 0.1$.

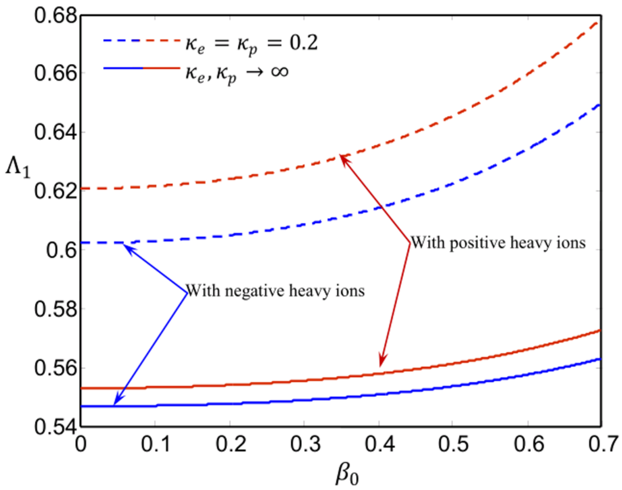


**Figure 4.** The variations of the growth rate $\Lambda_1$ versus the relativistic parameter $\beta_0$ in the presence of positive/negative heavy ions and for superthermal ($\kappa_j = 2$) and Maxwellian ($\kappa_j \rightarrow \infty$) electron-positron, with $\sigma_p = 1$, $\sigma = 0.01$, $\delta = 0.5$, $\alpha_h = 0.1$, $U = 0.1$, and $\Omega_c = 0.1$.

**ing positive heavy ions and the other containing negative heavy ions**). It is clear that the values of the instability growth rate $\Lambda_1$ are enhanced with increasing positron concentration $\delta$ for both the models. It was also observed that the value of $Lambda_1$becomes lower for the plasma model containing negatively charged heavy ions. The impact of the ion temperature (via the $\sigma$-parameter) on the instability growth rate $\Lambda_1$ is depicted in Fig. 6 for both models. The figure shows that the growth rate $\Lambda_1$ increaseds linearly with $\sigma$. However, the values of $\Lambda_1$ decrease for the plasma model containing negatively charged heavy ions (solid curve).

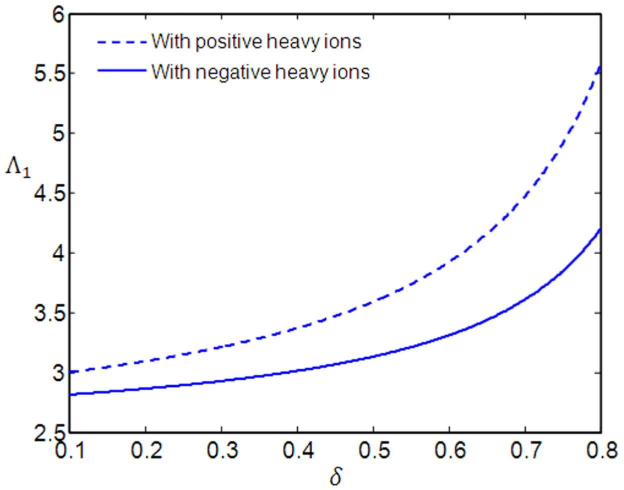


**Figure 5.** The variations of the growth rate $\Lambda_1$ versus the positron concentration $\delta$, for two plasma models: One with positive heavy ions (dashed curve) and the other with negative heavy ions (solid curve), along with $\kappa = 1.8$, $\alpha_h = 0.3$, $\beta_0 = 0.4$, $U = 0.5$, $\Omega_c = 0.1$, and $\sigma = 0.01$

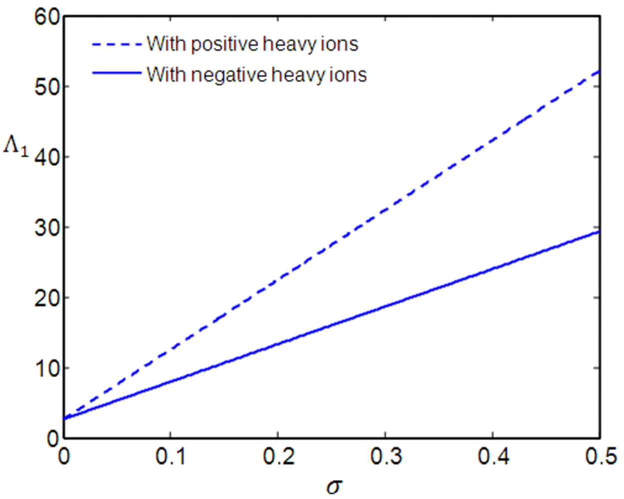


**Figure 6.** The variation of the growth rate $\Lambda_1$ versus the ion temperature $\sigma$, for two plasma models: One with positive heavy ions (dashed curve) and the other with negative heavy ions (solid curve), along with $\kappa = 1.8$, $\alpha_h = 0.3$, $\beta_0 = 0.4$, $U = 0.5$, $\Omega_c = 0.1$, and $\sigma = 0.01$

## 6. CONCLUSION

In this paper, we have investigated the instability of IA solitons propagating in a collisionless, magnetized plasma system composed of relativistic streaming light ions $(H^+)$, superthermally distributed electrons and positrons, in the presence of positive/negative heavy ion populations. Here, the heavy ions are assumed to be stationary background due to its massive mass compare to protons or electrons. According to the RPT, the two-dimensional ZK equation is derived, and its planar soliton solution is also obtained. Using the small-k perturbation expansion method, the instability of the plan soliton solution of the 2D ZK equation was analyzed to deduced the expression for the instability growth rate $\Lambda_1$. The effects of the plasma parameters, such as the relativistic parameter $\beta_0$, magnetic field strength (via $\Omega_c$), negative/positive heavy ion concentrations (via $\alpha_h$-parameter), positron concentration (via $\delta$-parameter), spectral index ($\kappa_j$), and ion temperature ratio (via $\sigma$-parameter) on the first-order instability growth rate $\Lambda_1$ have been investigated. It has been found that the stability of IA solitons is significantly modified by those parameters. The results also confirmed that the presence of negatively charged heavy ions in the plasma model can lead to stable solitons. The growth rate of the instability decreased as the magnetic field increased. The results also indicate that the stability of IA solitons is monotonically increases as a function of the spectral index ($\kappa$) and approaches the Maxwellian values $\kappa_j \to \infty$. Such plasma can be considered a simplified model for astrophysical plasmas, where relativistic streaming ions and superthermally distributed electrons and positrons coexist with heavy ion populations.

## REFERENCES

[1] H. R. Miller and P. J. Witter, *Active Galactic Nuclei*. Berlin: Springer, 1987.

[2] E. Tandberg-Hansen and A. G. Emslie, *The Physics of Solar Flares*. Cambridge: Cambridge University Press, 1988.

[3] A. V. Gurevich and Y. Istomin, *Physics of the Pulsar Magnetosphere*. Cambridge: Cambridge University Press, 1993.

[4] P. O. Dovner, A. I. Eriksson, R. Boström, and B. Holback, “Freja multiprobe observations of electrostatic solitary structures,” *Geophys. Res. Lett.*, vol. 21, pp. 1827–1830, 1994.

[5] P. K. Prasad and A. Saha, “Dynamical behavior and multistability of ion-acoustic waves in a magnetized auroral zone plasma,” *J. Astrophys. Astron.*, vol. 42, p. 9, 2021.

[6] S. I. Popel, S. V. Vladimirov, and P. K. Shukla, “Ion-acoustic solitons in electron-positrons-ion plasmas,” *Phys. Plasmas*, vol. 2, pp. 716–719, 1995.

[7] S. Mahmood and N. Akhtar, “Ion acoustic solitary waves with adiabatic ions in magnetized electron-positron-ion plasmas,” *Eur. Phys. J. D*, vol. 49, pp. 217–222, 2008.

[8] H. R. Pakzad, “Ion acoustic solitary waves in plasma with nonthermal electron and positron,” *Phys. Lett. A*, vol. 373, pp. 847–850, 2009.

[9] T. K. Baluku and M. A. Hellberg, "Ion acoustic solitary waves in an electron-positrons-ions plasma with non-thermal electrons," *Plasma Phys. Control. Fusion*, vol. 53, p. 095 007, 2011.

[10] A. E. Mowafy, "Nonlinear ion-acoustic solitary waves in electron-positron-ion inhomogenous plasma," *Theor. Math. Phys.*, vol. 71, pp. 229–236, 2016.

[11] N. Gul and R. Ahmad, "Dissipative ion-acoustic solitons in electron-positrons-ion plasma with non-thermal electrons and iso-thermal positrons," *Adv. Space Res.*, vol. 68, no. 1, pp. 161–169, 2021.

[12] B. Abraham-Shrauner, J. R. Asbridge, S. J. Bame, and W. C. Feldman, "Proton-driven electro-magnetic instabilities in high-speed solar wind streams," *J. Geophys. Res. Space Phys.*, vol. 84, pp. 553–559, 1979.

[13] A. T. Y. Lui and S. M. Krimigis, "Earthward transport of energetic protons in the earth's plasma sheet," *Geophys. Res. Lett.*, vol. 8, pp. 527–530, 1981.

[14] M. R. Collier and D. C. Hamilton, "The relationship between kappa and temperature in energetic ion spectra at jupiter," *Geophys. Res. Lett.*, vol. 22, no. 3, pp. 303–306, 1995.

[15] T. P. Armstrong, M. T. Paonessa, E. V. Bell, and S. M. Krimigis, "Voyager observations of saturnian ion and electron phase space densities," *J. Geophys. Res. Space Phys.*, vol. 88, no. A11, pp. 8893–8904, 1983.

[16] A. F. Vinas, H. K. Wong, and A. J. Klimas, "Generation of electron suprathermal tails in the upper solar atmosphere: Implications for coronal heating," *Astrophys. J.*, vol. 528, pp. 509–523, 2000.

[17] R. A. Shahein and M. H. Raddadi, "Bifurcation analysis of ion-acoustic solitary and periodic waves of superthermal electron-positron with a strongly relativistic plasma," *Appl. Math. Sci.*, vol. 16, no. 3, pp. 103–125, 2022.

[18] V. M. Vasyliunas, "Survey of low-energy electrons in the evening sector of the magnetosphere with ogo1 and ogo3," *J. Geophys. Res.*, vol. 73, no. 9, pp. 2839–2884, 1968.

[19] G. Williams and I. Kourakis, "On the existence and stability of electrostatic structures in non-maxwellian electron-positron-ion plasma," *Phys. Plasmas*, vol. 20, p. 122 311, 2013.

[20] M. Shahmansouri and E. Astaraki, "Transverse perturbation on three-dimensional ion-acoustic waves in electron-positron-ion plasma with high-energy tail electron and positron distribution," *J. Theor. Appl. Phys.*, vol. 8, pp. 189–201, 2014.

[21] J. Wang, Y. Zeng, Z. Liang, Y. Xu, and Y. Zhang, "Ion acoustic quasi-soliton in an electron-positron-ion plasma with superthermal electrons and positrons," *Open Phys.*, vol. 16, pp. 563–567, 2018.

[22] J. Ozah and P. N. Deka, "Dynamical behaviour of nonlinear structures in superthermal plasmas associated with external periodic force," *Indian J. Phys.*, vol. 97, pp. 2197–2208, 2023.

[23] S. Y. El-Monier, A. S. El-Helbawy, M. M. Elsayed, and M. Saad, "The role of superthermal electrons and positrons on magnetized oscillatory shock waves," *Pramana-J Phys.*, vol. 98, p. 13, 2024.

[24] S. J. Bame, J. R. Asbridge, W. C. Feldman, M. D. Montgomery, and P. D. Kearney, "Solar wind heavy ion abundances," *Sol. Phys.*, vol. 43, pp. 463–473, 1975.

[25] P. R. Young, "The ne/o abundance ratio in the quiet sun," *A@AND@A*, vol. 444, pp. L45–L48, 2005.

[26] C. M. S. Cohen, G. M. Mason, and R. A. Mewaldt, "Characteristics of solar energetic ions as a function of longitude," *ApJ*, vol. 843, no. 2, p. 132, 2017.

[27] M. A. Hossen, M. R. Hossen, and A. A. Mamun, "Modified ion-acoustic shock waves and double layers in a degenerate electron-positron-ion plasma in presence of heavy negative ions," *Braz. J. Phys.*, vol. 44, pp. 703–710, 2014.

[28] J. I. Vette, *Summary of Particle Population in the Magnetosphere.* Dordrecht: Reidel, 1970.

[29] H. Ikezi, "Experiments on ion-acoustic solitary waves," *Phys. Fluids*, vol. 16, pp. 1668–1675, 1973.

[30] J. Arons, "Some problems of pulsar physics," *Space Sci. Rev.*, vol. 24, pp. 437–510, 1979.

[31] T. S. Gill, A. S. Bains, and N. S. Saini, "Ion acoustic soliton in weakly relativistic magnetized electron–positron–ion plasma," *Can. J. Phys.*, vol. 87, pp. 861–866, 2009.

[32] H. R. Pakzad, "Ion acoustic solitons of kdv and modified kdv equations in weakly relativistic plasma containing nonthermal electron, positron and warm ion," *Astrophys. Space Sci.*, vol. 332, pp. 269–277, 2011.

[33] M. A. Khaled, "On the head-on collision between two ion acoustic solitary waves in a weakly relativistic plasma containing nonextensive electrons and positrons," *Astrophys. Space Sci.*, vol. 350, pp. 607–614, 2014.

[34] M. G. Hafez and M. R. Talukder, "Ion acoustic solitary waves in plasmas with nonextensive electrons, boltzmann positrons and relativistic thermal ions," *Astrophys. Space Sci.*, vol. 359, pp. 27–39, 2015.

[35] K. Javidan and D. Saadatmand, "Effect of high relativistic ions on ion-acoustic solitons in electrons-positrons-ions plasmas with nonthermal electrons and thermal positrons," *Astrophys. Space Sci.*, vol. 333, pp. 471–475, 2011.

[36] K. Javidan and H. R. Pakzad, "Ion acoustic solitary waves in high relativistic plasmas with superthermal electrons and thermal positrons," *Indian J. Phys.*, vol. 86, no. 11, pp. 1037–1042, 2012.

[37] J. Kalita, R. Das, K. Hosseini, D. Baleanu, and E. Hincal, "Ion acoustic soliton with thermal ions and non-thermal electrons in a high-relativistic electron-positron-ion plasma," *Partial. Differ. Equ. Appl. Math.*, vol. 8, p. 100 579, 2023.

[38] K. Jyotishmita, M. Bhargab, and D. Ranjan, "High-relativistic effect on ion acoustic solitons in electron-positron-ion plasma," *J. Korean Phys. Soc.*, vol. 84, no. 2, pp. 120–127, 2024.

[39] A. Mushtaq and H. A. Shah, "Nonlinear zakharov–kuznetsov equation for obliquely propagating two-dimensional ion-acoustic waves in a relativistic, rotating magnetized electron–positron–ion plasma," *Phys. Plasmas*, vol. 12, no. 7, p. 072 306, 2005.

[40] S. Raut, K. K. Mondal, P. Chatterjee, and A. Roy, "Two-dimensional ion-acoustic solitary waves obliquely propagating in a relativistic rotating magnetized electron-positron-ion plasma in the presence of external periodic force," *Pramana-J. Phys.*, vol. 95, no. 1, pp. 73–85, 2021.

[41] J. Ozah and P. N. Deka, "Dynamics of ion-acoustic waves in a magnetized plasma with two-temperature superthermal electrons," *ICNDA*, vol. 405, pp. 370–381, 2024.

[42] M. A. Allen and G. Rowlands, "Determination of the growth rate for the linearized zakharov-kuznetsov equation," *J. Plasma Phys.*, vol. 50, pp. 413–424, 1993.

[43] M. A. Allen and G. Rowlands, "Stability of obliquely propagating plane solitons of the zakharov-kuznetsov equation," *J. Plasma Phys.*, vol. 53, pp. 63–73, 1995.

[44] M. Shalaby, S. K. EL-Labany, E. F. EL-Shamy, W. F. El-Taibany, and M. A. Khaled, "On the stability of obliquely propagating dust ion-acoustic solitary waves in hot adiabatic magnetized dusty plasmas," *Phys. Plasmas*, vol. 16, p. 123 706, 2009.

[45] M. Shalaby, S. K. EL-Labany, E. F. EL-Shamy, and M. A. Khaled, "Three-dimensional instability of dust ion-acoustic solitary waves in a magnetized dusty plasma with two different types of nonisothermal electrons," *Phys. Plasmas*, vol. 17, p. 113 709, 2010.

[46] M. A. H. Khaled, "Dust ion acoustic solitary waves and their multi-dimensional instability in a weakly relativistic adiabatic magnetized dusty plasma with two different types of adiabatic electrons," *Indian J. Phys.*, vol. 88, pp. 647–656, 2014.

[47] N. Akhtar et al., "Transverse instability of ion acoustic solitons in a magnetized plasma including q-nonextensive electrons and positrons," *J. Plasma Phys.*, vol. 81, p. 90 581 051, 2015.

[48] C. Liu, L. Wang, X. Yang, and Y. Shi, "Transverse instability of dust-acoustic solitary waves in magnetized dusty plasmas," *Plasma Sci. Technol.*, vol. 17, no. 4, pp. 298–302, 2015.

[49] N. A. Zedan, A. Atteya, W. F. El-Taibany, and S. K. EL-Labany, "Stability of ion-acoustic solitons in a multi-ion degenerate plasma with the effects of trapping and polarization under the influence of quantizing magnetic field," *Waves Random Complex Media*, vol. 32, no. 2, pp. 728–742, 2020.

[50] M. A. H. Khaled, M. A. Shukri, and R. A. A. Fare'e, "Modulated ion-acoustic wavepackets in a multi-component dense plasma with thomas-fermi distribution," *JAST*, vol. 1, no. 1, pp. 83–92, 2023.

[51] A. P. Misra and A. Abdikian, "Transverse instability of electron-acoustic solitons in a relativistic degenerate astrophysical magnetoplasma," *Phys. Scr.*, vol. 100, p. 015 601, 2025.

[52] H. Washimi and T. Taniuti, "Propagation of ion acoustic solitary waves of small amplitude," *Phys. Rev. Lett.*, vol. 17, pp. 996–998, 1966.