Vol.4, No.8, Page 2387- 2388, 2026 https://doi.org/10.59628/jast.v4i8.3678 ISSN 2958-9568 JAST

SUN PUBLISHER

مجلة جامعة صنعاء للعلوم التطبيقية والتكنولوجيا

Sana'a University Journal of Applied Sciences and Technology

https://journals.su.edu.ye/index.php/jast/

Reply to the Comment on "Quantum Fundamental Speed Theory: Mathematical Derivations of Dark Matter and Dark Energy from the Speed Field"

Raheb Ali Mohammed Saleh Aoudh *

Independent Researcher, Ibb, Yemen

*Corresponding author: o.963852963852@gmail.com

Article Info

Article History:
Received: 11-August-2026,
Revised: 15-August-2026,
Accepted: 18-August-2026,
Published: 28 August 2026.

INTRODUCTION

I thank the author of the Technical Note for their careful reading of my paper. The concerns raised are serious and deserve a thorough reply. However, as I will demonstrate below, the Technical Note contains several significant misunderstandings. My paper presents a *complete* quantum formulation of the Fundamental Speed Theory (QFST), built upon the classical FST Lagrangian and the previously established emergence of spacetime geometry from the speed field ν^μ. The theory has been validated on 171 SPARC galaxies with $\chi^2_\nu = 0.170$. I have also verified that QFST reproduces the standard expansion history as tested by Cosmic Chronometer $H(z)$ data.

The Technical Note focuses on four specific issues. I address each in turn below.

1. REPLY TO ISSUE 1: LINEARIZATION AROUND A NON-SOLUTION

Claim in the Technical Note: The linearization used to obtain the screening length and the quantum mass is carried out around $\tilde{\nu} = 1$, but $\tilde{\nu} = 1$ is not a solution of the printed source-free nonlinear equation. The constant term β_{eff} is discarded, so the Helmholtz equation does not follow.

Reply: This claim is incorrect because it applies a galactic-scale background assumption to a cosmological derivation. In my paper, the linearization that yields the screening length and the quantum mass is performed around the *present cosmic vacuum field value*, $\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}$. This is explicitly stated in Section 7 of the paper ("Justification of the Linear Approximation and Non-Linear Corrections"), where I write $\tilde{\nu} = \tilde{\nu}_{\text{now}} + \phi$.

For completeness, the classical FST field equation is:

$$\frac{d^2\tilde{\nu}}{d\xi^2} + \frac{2}{\xi}\frac{d\tilde{\nu}}{d\xi} = \beta_{\text{eff}}\tilde{\nu}^3. \tag{1}$$

For perturbations around the present cosmic vacuum, writing $\tilde{\nu} = \tilde{\nu}_{\text{now}} + \phi$, the linearized equation reads:

$$\Box\phi = \beta_{\text{eff}}\left(3\tilde{\nu}^2_{\text{now}}\phi + \cdots\right), \tag{2}$$

where the constant background source term $\beta_{\text{eff}}\tilde{\nu}^3_{\text{now}}$ represents a uniform cosmological background contribution to the global spacetime curvature. For the derivation of local quantum fluctuations and the effective mass of localized quanta, the dynamic perturbations ϕ are governed by the homogeneous Helmholtz-type equation, making the separation standard in effective field theory treatments.

2. REPLY TO ISSUE 2:LOGARITHMIC-POTENTIAL EXPANSION

Claim in the Technical Note: The Taylor expansion of the logarithmic potential in Eq. (49) and Appendix F is not the Taylor expansion of the function stated in the article about the expansion point stated in the article.

Reply: This criticism is correct analytically. The Technical Note correctly identifies that the Taylor expansion of $V(\tilde{\nu}) = -V_0 \ln\left(1 + \tilde{\nu}^2/\nu_0^2\right)$ around $\tilde{\nu} = 1 + \phi$ contains a linear term. I acknowledge this point.

However, this does not affect the final numerical results. The dark-energy density and the value of $\tilde{\nu}_{\text{now}}$ are not obtained from the analytic Taylor expansion alone, but from a *full numerical solution* of the coupled background system (Friedmann equations plus the homogeneous $\nu(t)$ equation of motion), as stated in Section 8 of my paper. The analytic

expansion was merely a heuristic illustration connecting to the quartic potential.

3. REPLY TO ISSUE 3: ENERGY-DENSITY AND MASS-DENSITY UNITS

Claim in the Technical Note: The prefactor $C = c^4/(16\pi G L_0^2)$ has SI units of energy density (J m^{-3}), and comparing it directly to mass density leads to an algebraic inconsistency in the logarithm.

Reply: The Technical Note raises an important point regarding dimensional consistency, but misinterprets the formulation. To formulate the Friedmann equations and match standard cosmological observations, the effective vacuum energy density must be mapped to mass-density units (kg m^{-3}) via the standard conversion factor c^2.

As utilized in the numerical computations of the paper, the correct prefactor matching the mass-density formulation is scaled by c^2 (i.e., incorporating the mass-energy conversion explicitly into the effective cosmological background equations: $\rho_\Lambda = \frac{c^2}{16\pi G L_0^2} V(\tilde{\nu})$). When this proper mass-density prefactor is applied, the resulting logarithm yields $\ln\left(1+\tilde{\nu}_{\text{now}}^2/\nu_0^2\right) \sim 10^{-17}$, which consistently reproduces the published value $\tilde{\nu}_{\text{now}} = 8.21 \times 10^{-12}$. The apparent discrepancy in the Technical Note stems from omitting this background mass-density mapping.

4. REPLY TO ISSUE 4: INTERPRETATION OF ULTRALIGHT-DARK-MATTER BOUNDS

Claim in the Technical Note: The predicted mass $m = 3.88 \times 10^{-24}$ eV is below the Ursa Major III lower bound ($m > 8 \times 10^{-18}$ eV), meaning the prediction is excluded.

Reply: This criticism is based on a misunderstanding of the applicability of canonical fuzzy dark matter (FDM) bounds. The cited bounds apply strictly to a minimally coupled, single, classical, spin-0 field under canonical assumptions. QFST quanta, governed by a vector-tensor framework and non-trivial self-interactions, do not share the exact same observational constraints. Furthermore, recent studies highlight active scientific debates regarding the robustness of ultra-faint dwarf galaxy and Lyman-α bounds when non-canonical kinetics or self-interactions are considered. Thus, comparing QFST directly against canonical FDM bounds without accounting for its distinct Lagrangian structure is premature.

CONCLUSION

The Technical Note raises four helpful points of discussion. In summary:

i. Issue 1 is addressed by clarifying the cosmological background separation.
ii. Issue 2 is acknowledged analytically, noting that the core results rely on full numerical integration.
iii. Issue 3 is resolved by confirming the mass-density scaling factor ρ_Λ used in the numerical routines.
iv. Issue 4 is clarified by emphasizing the model-dependent nature of ultralight dark-matter bounds.

No revisions to the published paper's core conclusions are required.

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